Software Engineering 3DX3

Slides 3: Reduction of Multiple Subsystems

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Material based on lecture notes by P. Taylor and M. Lawford, and *Control Systems Engineering* by N. Nise.

Introduction

- So far he have represented systems as a single block (transfer function), with its inputs and outputs.
- Many systems are much more complicated and represented by many interconnected subsystems.
- As its straightforward to calculate the response of a single transfer function, we want to be able to convert multiple subsystems into an equivalent single transfer function.
- We will use block diagram algebra to do the reduction.
- This then allows us to apply the techniques we have already developed to the resulting single subsystem.

Block Diagrams

- When interconnecting multiple subsystems, we need more elements than just a single block with inputs and outputs.
- We add the elements:

  Summing Junctions: they combine two or more signals, producing the algebraic sum as output.

Pickoff Points: it breaks input signal into multiple copies to be sent to different destinations.

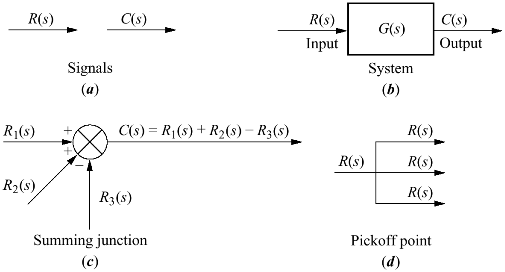


Figure 5.2

Cascade Form

- First common interconnection method we look at is called the cascade form.

- Consists of two or more subsystems connected in a serial fashion.

- Equivalent to a single block with transfer function equal to product of the individual block's transfer functions.

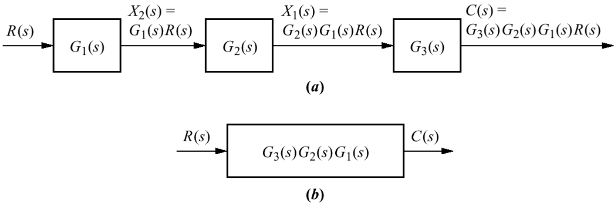


Figure 5.3.

Loading in Cascaded Subsystems

- Formula for combining cascaded subsystems is invalid when a given subsystem loads its preceding subsystem.
- A given subsystem is not loaded by the next subsystem if its output is unchanged by connecting the following subsystem.

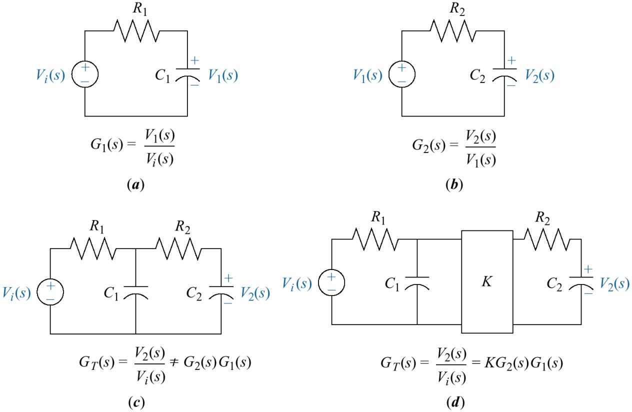


Figure 5.4.

Parallel Form

- For subsystems connected in parallel:
  - They all have same input.
  - The output of the group is the sum of each individual subsystem's output.
- The equivalent transfer function is the sum of the individual transfer functions.

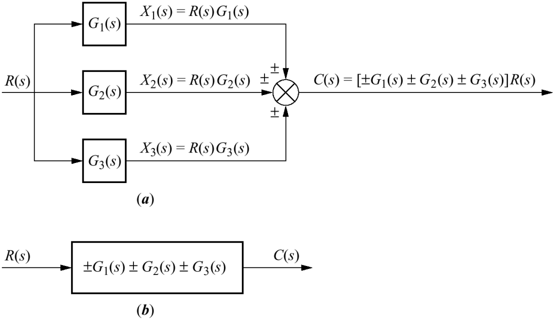


Figure 5.5.

Feedback Form

- Feedback topology is basis of control systems theory.
- In Simplified model (Fig. 5.6(b)), we see that:

$$E(s) = R(s) \mp C(s)H(s)$$

- We also see $C(s) = E(s)G(s)$ thus $E(s) = \frac{C(s)}{G(s)}$.
- Substituting in above gives: $G_e(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$
- We call $G(s)H(s)$ the open loop transfer function or loop gain.

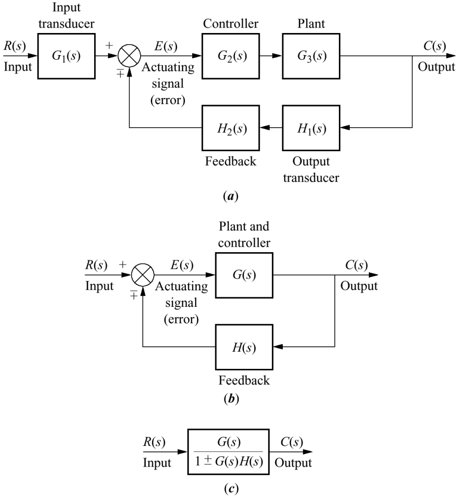


Figure 5.6.

Moving Blocks to Create Familiar Forms

- Have examined three different topologies so far.
- In physical systems, we will find them combined into complex arrangements.
- Recognizing these structures will be key to reducing more complex systems to a single transfer function.
- Unfortunately, these forms may be present, but not always obvious.
- We will learn how to move blocks forward or backwards past summing junctions and pickoff points.

Moving Blocks Through Summing Junctions

- Top figure shows the equivalent diagram when block moved to the left of junction.

- Can see they are equivalent by noting that on left, $C(s) = [R(s) \mp X(s)]G(s) = R(s)G(s) \mp X(s)G(s)$.

- Bottom figure shows equivalent system when moving block to the right of junction.

- Can see equivalent since on right,

$$\begin{aligned} C(s) &= [R(s) \mp \frac{X(s)}{G(s)}]G(s) \\ &= R(s)G(s) \mp X(s) \end{aligned}$$

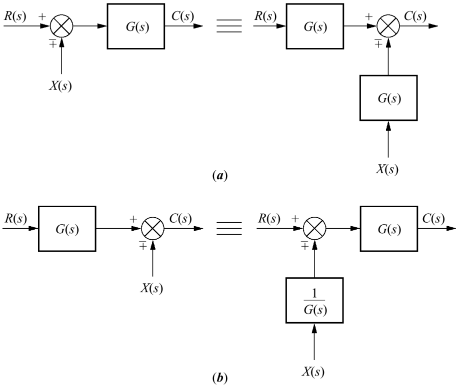


Figure 5.7.

Moving Blocks Through Pickoff Points

- Top figure shows the equivalent diagram when block moved to the left of pickoff point.
- Bottom figure shows equivalent system when moving block to the right of pickoff point.

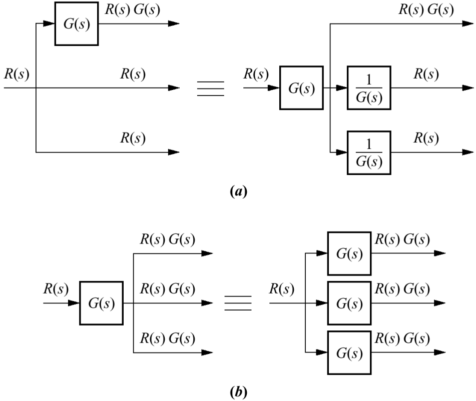


Figure 5.8.

Reduction Via Familiar Forms eg.

- Reduce block diagram to a single transfer function.

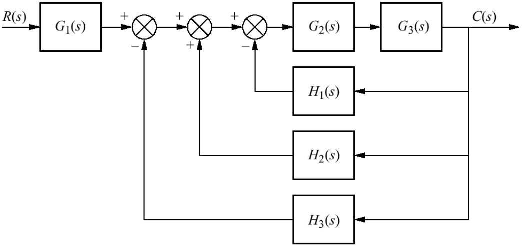


Figure 5.9.

- We start by noting that the three summations are just doing algebraical sums and can be combined

Reduction Via Familiar Forms eg. - II

- Combining the summations gives Fig. (a).
- Applying parallel and cascade rule gives Fig. (b).
- Applying feedback rule, followed by cascade rule to combine with $G_1(s)$, gives Fig. (c).

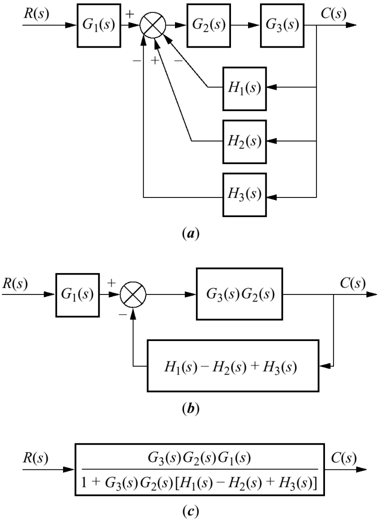


Figure 5.10.

Reduction by Moving Block eg.

- Reduce block diagram to a single transfer function.

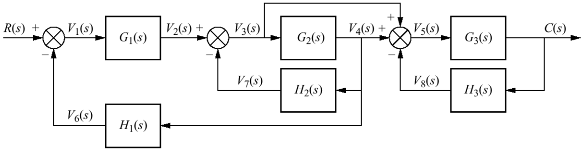


Figure 5.11.

1. Move G_2 to left of pickoff point creating parallel form.
2. Reduce feedback system (G_3, H_3).

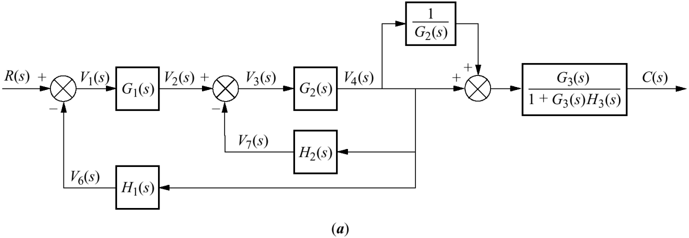
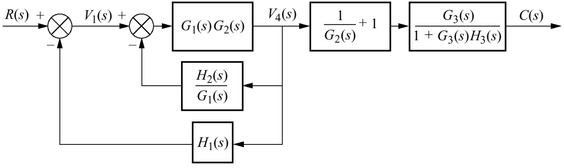


Figure 5.12.

Reduction by Moving Block eg. - II

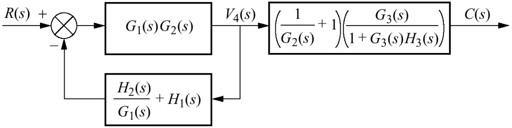
3. Reduce parallel form containing $\frac{1}{G_2(s)}$ and unity.
4. Push $G_1(s)$ to the right past summing junction. Creates parallel form (H_1 and $[\frac{1}{G_1}, H_2]$).
5. Combine serial forms (G_1, G_2) and ($\frac{1}{G_1}$, H_2).



(b)

Figure 5.12.

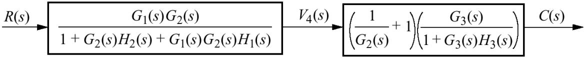
6. Collapse summing junctions, and combine parallel form.
7. Combine serial form on right.



(c)

Reduction by Moving Block eg. - III

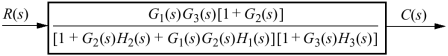
8. Collapse feedback form.



(*d*)

Figure 5.12.

9. Combine the two cascade blocks.



(*e*)

Figure 5.12.