# Software Engineering 3DX3

# Slides 5: Time Response

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Material based on lecture notes by P. Taylor and M. Lawford, and Control Systems Engineering by N. Nise.

# Introduction

- Once we have obtained a mathematical representation of a system, our next step is to analyze its transient and steady-state response.
- In this section, we will focus on how to analyze a system's transient response.
- We already know how to determine the output response by solving differential equations, or taking inverse Laplace transforms.
- > These methods are laborous and time consuming.
- We want to develop a technique where we can get the desired information about a systems transient and steady-state response, basically by inspection.
- Our first topic will be how to analyze poles and zeros to determine a system's response.

#### **Poles and Zeros**

► Consider:

$$G(s) = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \dots + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_0} = \frac{N(s)}{D(s)}$$

- Poles of G(s) are the roots of D(s).
- Zeros of G(s) are the roots of N(s).
- Generally, at poles G(s) = ∞ unless the pole is cancelled by a matching zero.
- At zeros, G(s) = 0 unless the zero is cancelled by a matching pole.

### Poles and Zeros of First Order System eg.

A system's output response contains two parts:

- 1. Forced or steady state response: this is caused by the poles of the input function, R(s).
- 2. Natural or homogeneous response: this is caused by the poles of the transfer function, G(s).

► In example below, our transfer function is G(s) = <sup>s+2</sup>/<sub>s+5</sub>, and our input is R(s).



Figure 4.1.

#### Poles and Zeros of First Order System eg. - II





### Poles and Zeros of First Order System eg. - III

- **1.** Pole of input function generated forced response, u(t).
- 2. Pole of transfer function generated natural response,  $e^{-5t}$ .

The above is not affected at all by the zero.

3. Pole on real axis, say at  $-\alpha$ , generates an exponential response,  $e^{-\alpha t}$ .

Farther to the left on negative axis, the faster the response decays.

4. Both the poles and zeros contribute to the amplitude of the response (ie. the  $\frac{2}{5}$  and  $\frac{3}{5}$  factors).



### Evaluating Response using Poles eg.

Consider system shown below.

From inspection, we can immediately determine:





# First Order Systems

- We will now examine first order systems without zeros so we can define performance specifications.
- We use systems of the form G(s) = <sup>a</sup>/<sub>s+a</sub> as our base form for our definitions.

• If our input is the step function,  $R(s) = \frac{1}{s}$ , we get

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$



Figure 4.4.

# **Time Constant**

- We will now examine first order systems without zeros so we can define performance specifications.
- Our first specification is the system's time constant,  $\tau = \frac{1}{a}$ .
- The time constant is the time required for step response to rise to 63% percent of its final value.

$$c(\tau) = 1 - e^{-a\tau} = 1 - 0.37 = 0.63$$

As 
$$\frac{dc(t)}{dt} = ae^{-at}$$
, we thus have a equal to the slope at  $t = 0$ .

We call a the exponential frequency.

Figure 4.5



### **Rise and Settling Time**

• Rise time,  $T_r$ , is the time for the output to go from 10% to 90% of its final value.

• Can show that 
$$T_r = \frac{2.2}{a}$$
.

• Settling time,  $T_s$ , is time required for the output to reach 98% of its final value.

• Setting 
$$c(T_s) = 0.98$$
, we find that  $T_s = \frac{4}{a}$ .

Figure 4.5



### Using Testing to Determine Transfer Function

- It is quite often not possible or practical to determine a systems transfer function by analytical means.
- ▶ In general, gain of system at s = 0 (D.C. gain) is not unity.

• A more general model would be  $G(s) = \frac{K}{s+a}$ .

Step response is thus

$$C(s) = \frac{K}{s(s+a)} = \frac{\frac{K}{a}}{s} - \frac{\frac{K}{a}}{s+a}$$

have 
$$c(t) = rac{K}{a}(1-e^{-at})$$

▶ How can we experimentally determine the values of *K* and *a*?

We thus

# Using Testing to Determine Transfer Function - II

- For system to be a first-order system, its unit step response should have no overshoot and should have a nonzero initial slope, as in diagram below.
- ▶ From diagram, we can see the final value is about 0.72, thus 63% of that is 0.63 × 0.72 = 0.45.
- From diagram, the output reaches 0.45 at about \(\tau = 0.13\) (time constant).

• We thus have 
$$a = \frac{1}{\tau} = 7.7$$
.

▶ We next note that  $c(\infty) = \frac{K}{a}(1 - e^{-at})|_{t \to \infty} = \frac{K}{a}$ ▶ Thus  $K = a \cdot c(\infty) = (7.7)(0.72) = 5.54$ 

• Thus 
$$G(s) = \frac{5.54}{s+7.7}$$



Figure 4.6

# **Second-Order Systems**

- For first order systems, varying the systems parameters only changed the speed of the response.
- Form of a second order system we will analyze is  $G(s) = \frac{b}{s^2 + as + b}$ .
- Changes in these parameters can actually change the form of the system's response.
- May see responses similar to first-order system, damped oscillations, or undamped oscillations.

# Second-Order System Examples



Figure 4.7.

# Second-Order System Examples -II





 Like for first-order system, we want to determine information about system's steady state and transient response by examination.

### **Overdamped Response**

For overdamped response, we have a system with two non equal real poles.

The unit step response to system below is

$$C(s) = \frac{9}{s(s^2 + 9s + 9)} = \frac{9}{s(s + 7.854)(s + 1.146)}$$

From inspection of poles, we know form of system's response will be:

$$c(t) = K_1 + K_2 e^{-\sigma_1 t} + K_3 e^{-\sigma_2 t}$$

where  $-\sigma_1 = -7.854$  and  $-\sigma_2 = -1.146$ , are our two real poles.



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Figure 4.7.

# **Critically Damped Response**

For critally damped response, we have a system with two equal real poles.

The unit step response to system below is

$$C(s) = \frac{9}{s(s^2 + 6s + 9)} = \frac{9}{s(s+3)^2} = \frac{K_1}{s} + \frac{K_2}{(s+3)} + \frac{K_3}{(s+3)^2}$$

From inspection of poles, we know form of system's response will be:

$$c(t) = K_1 + K_2 e^{-\sigma_1 t} + K_3 t e^{-\sigma_1 t}$$

where  $-\sigma_1 = -3$  is our pole location.



Figure 4.7.

### **Underdamped Response**

- For underdamped response, we have a system with two complex conjugate poles (non zero real and imaginary parts).
- The unit step response to system below is

$$C(s) = \frac{9}{s(s^2 + 2s + 9)} = \frac{9}{s(s + 1 + j\sqrt{8})(s + 1 - j\sqrt{8})}$$
$$= \frac{K_1}{s} + \frac{\alpha + j\beta}{s + 1 + j\sqrt{8}} + \frac{\alpha - j\beta}{s + 1 - j\sqrt{8}}$$

Thus the form of system's response will be:

$$c(t) = K_1 + e^{-\sigma_d t} [2\alpha \cos \omega_d t + 2\beta \sin \omega_d t]$$
  
where  $-\sigma_d \pm j\omega_d = -1 \pm j\sqrt{8}$ .



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Figure 4.7.

### Underdamped Response - II

- For system with poles at s = −σ<sub>d</sub> ± jω<sub>d</sub>, the real part (σ<sub>d</sub>) determines the exponential frequency (decay rate) for the exponential envelope.
- The imaginary part, ω<sub>d</sub>, determines the oscillation frequency of the sinusoids, and is called the damped frequency of oscillation.
- Can show that

$$e^{-\sigma_d t} [2\alpha \cos \omega_d t + 2\beta \sin \omega_d t]$$
  
=  $K_4 e^{-\sigma_d t} \cos(\omega_d t - \phi)$ 

where 
$$\phi = \tan^{-1}(\frac{\beta}{\alpha})$$
 and  $K_4 = \sqrt{(2\alpha)^2 + (2\beta)^2}$ 





### **Undamped Response**

 For undamped response, we have a system with two imaginary poles (zero real part).

The unit step response to system below is

$$C(s) = \frac{9}{s(s^2+9)} = \frac{9}{s(s+j3)(s-j3)} = \frac{K_1}{s} + \frac{\alpha+j\beta}{s+j3} + \frac{\alpha-j\beta}{s-j3}$$

Thus the form of system's response will be:

$$c(t) = K_1 + e^{-(0)t} [2\alpha \cos \omega_d t + 2\beta \sin \omega_d t]$$
  
=  $K_1 + 2\alpha \cos \omega_d t + 2\beta \sin \omega_d t$   
=  $K_1 + K_4 \cos(\omega_d t - \phi)$ 



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# Second-order System's Step Responses

- Critically damped case represents the transition between the underdamped and overdamped cases.
- Critically damped case is the fastest response without overshoot.



# **General Second-Order Systems**

- We now generalize our discussion of second-order systems and develop specifications to describe the response of the system.
- 1. The **natural frequency**,  $\omega_n$ , of a second-order system is the frequency of oscillation of the system with damping removed.
- 2. The damping ratio of a second-order system is a way to describe a system's damped oscillation, independent of time scale.

We define *damping ratio*,  $\zeta$ , to be

 $\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency(rad/sec)}}$ 

#### **Deriving Parameters** - $\omega_n$

 $\blacktriangleright$  We want to rewrite the second-order system shown below in terms of  $\omega_n$  and  $\zeta$ 

$$G(s) = \frac{b}{s^2 + as + b} \tag{1}$$

The quadratic equation tells us the poles are:

$$s_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2}$$
(2)

▶ To determine  $\omega_n$ , we need an undamped system; thus a = 0,  $G(s) = \frac{b}{s^2+b}$ .

• Our poles are thus  $s_{1,2} = \pm j\sqrt{b}$ , giving  $\omega_n = \sqrt{b}$  and  $b = \omega_n^2$ .

### **Deriving Parameters** - $\zeta$

$$G(s) = \frac{b}{s^2 + as + b} \qquad \qquad s_{1,2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2}$$

- For an underdamped system, the poles must have a real part,  $\sigma = \frac{-a}{2}$ .
- The *exponential decay frequency* is equal to the absolute value of σ.

$$\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency(rad/sec)}} = \frac{|\sigma|}{\omega_n} = \frac{a/2}{\omega_n} \qquad (3)$$

We thus have:

$$a = 2\zeta\omega_n \tag{4}$$

We can now rewrite our system as

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{5}$$

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### **Relating Parameters to Poles**

Substituting into the poles equation gives:

$$s_{1,2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2} = \frac{-(2\zeta\omega_n)}{2} \pm \frac{\sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$
(6)



#### **Relating Parameters to Poles - II**







### **Underdamped Second-order Systems**

- We are mostly interested in underdamped systems as they give us the fastest response.
- Need to examine behavior more closely for analysis and design.
- Will now define transient specifications for underdamped responses.
- The step response is:

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
(7)

 Assuming ζ < 1 (ie. underdamped case), partial fractions gives:

$$C(s) = \frac{1}{s} - \frac{(s + \zeta\omega_n) + \frac{\zeta}{\sqrt{1 - \zeta^2}}\omega_n\sqrt{1 - \zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}$$
(8)

#### Underdamped Second-order Systems - II

$$C(s) = \frac{1}{s} - \frac{(s + \zeta\omega_n) + \frac{\zeta}{\sqrt{1 - \zeta^2}}\omega_n\sqrt{1 - \zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}$$

We can now use the inverse Laplace transform below that we derived earlier

$$\mathcal{L}\{K_1e^{-at}\cos\omega t + K_2e^{-at}\sin\omega t\} = \frac{K_1(s+a) + K_2\omega}{(s+a)^2 + \omega^2}$$

This gives:

$$c(t) = 1 - e^{-\zeta\omega_n t} (\cos\omega_n \sqrt{1 - \zeta^2} t + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin\omega_n \sqrt{1 - \zeta^2} t)$$

$$=1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos(\omega_n\sqrt{1-\zeta^2}\,t-\phi) \tag{9}$$

where 
$$\phi = \tan^{-1}(\frac{\zeta}{\sqrt{1-\zeta^2}})$$
.

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#### Underdamped Second-order Systems - III

$$c(t) = 1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \cos(\omega_n \sqrt{1 - \zeta^2} t - \phi)$$

where  $\phi = \tan^{-1}(\frac{\zeta}{\sqrt{1-\zeta^2}}).$ 

We can now plot the output with the time axis normalized to the natural frequency.



# **Underdamped Response Specifications**

Let  $c_{final} = \lim_{t \to \infty} c(t)$ .

- 1. Rise time,  $T_r$ , is the time for the output to go from 10%  $(0.1c_{final})$  to 90%  $(0.9c_{final})$  of its final value.
- **2.** Peak time,  $T_p$ , is the time required to reach the first and largest peak,  $c_{max}$ .
- 3. Percent overshoot, %OS, is the percentage that the output overshoots the final value at  $t = T_p$ .

$$OS = rac{c_{max} - c_{final}}{c_{final}} imes 100\%$$

4. Settling time,  $T_s$ , is time required for the output to reach and stay within  $\pm 2\%$  of

```
c_{final}.
```

Figure 4.14



#### **Calculating Peak time**

• We can determine  $T_p$  by differentiating c(t) in equation 9 and finding the first time it equals zero after t = 0.

$$\mathcal{L}\{\dot{c}(t)\} = sC(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$
(10)

As we want in form of L{sinwt}, we complete the square for the denominator giving

$$\mathcal{L}\{\dot{c}(t)\} = sC(s) = \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} = \frac{\frac{\omega_n}{\sqrt{1 - \zeta^2}} \omega_n \sqrt{1 - \zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}$$
(11)

We thus have

$$\dot{c}(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t)$$
(12)

#### Calculating Peak time - II

• Setting  $\dot{c}(t) = 0$  gives

$$\omega_n \sqrt{1 - \zeta^2} t = n\pi$$

thus

$$t = \frac{n\pi}{\omega_n \sqrt{1 - \zeta^2}} \tag{13}$$

At n = 1, we get first time derivative equals zero after t = 0. We thus have:

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \tag{14}$$

# **Calculating %OS**

The definition of percent overshoot is:

$$OS = rac{c_{max} - c_{final}}{c_{final}} imes 100\%$$

► To determine c<sub>max</sub>, we need to evaluate c(T<sub>p</sub>) by substituting equation 14 into equation 9:

$$c(t) = 1 - e^{-\zeta \pi / \sqrt{1 - \zeta^2}} (\cos \pi + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \pi)$$
  
$$c(t) = 1 + e^{-\zeta \pi / \sqrt{1 - \zeta^2}}$$
(15)

- From equation 9, it is easy to see that  $c(\infty) = c_{final} = 1$ .
- Substituting into the %OS formula gives:

$$\text{\%OS} = \frac{1 + e^{-\zeta \pi / \sqrt{1 - \zeta^2}} - 1}{1} \times 100\%$$
  
$$\text{\%OS} = e^{-\zeta \pi / \sqrt{1 - \zeta^2}} \times 100\%$$
 (16)

### Calculating %OS -II

$$OS = e^{-\zeta \pi / \sqrt{1 - \zeta^2}} \times 100\%$$

- However, what if we knew which %OS we wanted?
- We can use the equation above to solve for ζ in terms of %OS. This gives:

$$\zeta = \frac{-\ln(\% OS/100)}{\sqrt{\pi^2 + \ln^2(\% OS/100)}}$$
(17)

# Calculating 2% Settling Time

- Settling time is when the ouput reaches and stays within 2% of its final value.
- This occurs at latest when the exponential envelope of equation 9 reaches the value of 0.02. This gives:

$$\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t} = 0.02$$
 (18)

Solving for t gives:

$$e^{-\zeta\omega_n t} = 0.02\sqrt{1-\zeta^2}$$
  

$$-\zeta\omega_n t = \ln(0.02\sqrt{1-\zeta^2})$$
  

$$T_s = \frac{-\ln(0.02\sqrt{1-\zeta^2})}{\zeta\omega_n}$$
  

$$T_s \approx \frac{4}{\zeta\omega_n}$$
(19)

# **Calculating Rise Time**

- There does not exist a precise relationship between rise time and damping ratio.
- ▶ Instead, we use a computer and equation 9 to solve for  $c(\omega_n t_1) = 0.1c_{final}$  and  $c(\omega_n t_2) = 0.9c_{final}$ , normalizing for the natural frequency.
- We then calculate the normalized rize time,  $\omega_n T_r$ , as  $\omega_n T_r = (\omega_n t_2) (\omega_n t_1)$
- ► Then, we can use charts below to solve for  $T_r$  given a specific  $\zeta$  and  $\omega_n$ .



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# System Response with Additional Poles

- In last section, we analyzed second-order systems.
- The formulas we have derived for percent overshoot, settling time, and peak time are only directly valid for systems with two complex poles and no zeros.
- However, sometimes we can approximate a higher-order system as a second-order system containing the dominant poles.
- > The dominant poles are the two poles farthest to the right.



Figure 4.23.

# System Response with Additional Poles - II

- How far away do the additional poles have to be?.
- Depends on the accuracy you want.
- Text assumes that if a pole is five times more to the left than the dominant poles, then system is represented by the dominant poles.
- ▶ If above met, you would design using the second-order approximation, then simulate final system to make sure it satisfies the design specifications such as %OS, and *T<sub>s</sub>* etc.

#### **Example Three Pole Systems**

Compare step responses of systems below:

$$T_1(s) = \frac{24.542}{s^2 + 4s + 24.542}$$
$$T_2(s) = \frac{24.542}{(s+10)(s^2 + 4s + 24.542)}$$
$$T_3(s) = \frac{24.542}{(s+3)(s^2 + 4s + 24.542)}$$



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Figure 4.24.

# **Justification for Ignoring Nondominant Poles**

- As long as the nondominant pole is far enough to the left, then its contribution to the output will be negligible.
- Easy to see that this will cause it to decay quickly, but what about its amplitude?
- Consider the third order system below

$$C(s) = \frac{bc}{s(s^2 + as + b)(s + c)} = \frac{A}{s} + \frac{Bs + C}{s^2 + as + b} + \frac{D}{s + c}$$

► If we assume steady state response is unity, and that the nondominant pole is at s = -c, we can then solve for the following constants using partial fractions:

$$A = 1; B = \frac{ca - c^2}{c^2 + b - ca} C = \frac{ca^2 - c^2 a - bc}{c^2 + b - ca}; D = \frac{-b}{c^2 + b - ca} (21)$$

~

# Justification for Ignoring Nondominant Poles - II

A = 1;  

$$B = \frac{ca - c^{2}}{c^{2} + b - ca}$$

$$C = \frac{ca^{2} - c^{2}a - bc}{c^{2} + b - ca};$$

$$D = \frac{-b}{c^{2} + b - ca}$$

• If we let  $c \to \infty$ , we find:

$$A = 1;$$
  $B = -1$   
 $C = -a;$   $D = 0.$  (22)

### System Response with Zeros

- ▶ We now examine systems with Zeros.
- As we saw before, zeros don't change the type of system, but can affect the constants found during partial fraction expansion.
- Consider the system below

$$G(s) = \frac{(s+a)}{(s+b)(s+c)} = \frac{A}{s+b} + \frac{B}{s+c}$$

where by partial fractions we have:

$$A = \frac{-b+a}{-b+c}; \qquad \qquad B = \frac{-c+a}{-c+b} \qquad (23)$$

#### System Response with Zeros -II

$$A = \frac{-b+a}{-b+c}; \qquad \qquad B = \frac{-c+a}{-c+b}$$

When the zero is far to the left, it will be much larger than the poles, thus:

$$A \approx \frac{a}{-b+c};$$
  $B \approx \frac{a}{-c+b}$ 

Our system then becomes

$$G(s) \approx a \left[ \frac{\frac{1}{-b+c}}{s+b} + \frac{\frac{1}{-c+b}}{s+c} \right] = \frac{a}{(s+b)(s+c)}$$

## **Nonminium-Phase System**

- What if the zero is in the right half plane (ie. a < 0)?
- If C(s) is the response of a system, then after adding zero at -a, we get:

$$(s+a)C(s) = sC(s) + aC(s)$$

- $\blacktriangleright$  If the derivative term sC(s) is larger than the scaled response aC(s), the system will initially follow the derivative in the wrong direction!
- If we take r(t) = -u(t) as our input, we could get a system like the one below.



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Figure 4.26.

#### **Pole Zero Cancellations**

Consider the system below:

$$G(s) = \frac{K(s+z_1)}{(s+p_1)(s^2+as+b)}$$
(24)

If z<sub>1</sub> and p<sub>1</sub> are close enough to each other, they can effectively cancel each other even though they are not exactly equal.

$$C(s) = \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)}$$
(25)  
=  $\frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} + \frac{-0.033}{s+4.01}$ (26)  
 $\approx \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6}$ (27)

# Analysis and Design of Feedback Systems

- Using the block diagram algebra we developed earlier, we can now apply our second-order system results to feedback systems.
- Applying feedback reduction, we find that the equivalent closed-loop transfer function of system on right is

$$T(s) = \frac{K}{s^2 + as + K} \tag{28}$$



Figures 5.6 and 4.14.

### Analysis and Design of Feedback Systems - II

$$T(s) = \frac{K}{s^2 + as + K}$$

As we increase K from zero, the poles of the system will go from overdamped (0 ≤ K < a<sup>2</sup>/4), critically damped (K = a<sup>2</sup>/4), to underdamped (K > a<sup>2</sup>/4).

$$s_{1,2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4K}}{2} \ s_{1,2} = \frac{-a}{2} \ s_{1,2} = \frac{-a}{2} \pm \frac{j\sqrt{4K - a^2}}{2}$$

# Gain Design for Transient Response

- Design the value for system gain ,K, such that the system response has 10% overshoot.
- Applying feedback reduction, our closed-loop transfer function becomes

$$T(s) = \frac{K}{s^2 + 5s + K}$$
(29)  

$$\xrightarrow{R(s) + \bigotimes_{-K}} \underbrace{K}_{s(s+5)} \underbrace{C(s)}_{-K}$$
Figure 5.16.