Solving the Bloch Equations: Application to the Hahn Echo

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No Pulse is Perfect

•Finite width, finite pulse power
•Must account for evolution and relaxation during pulse
• Solve Bloch equations exactly
•Test with Hahn echo experiments for T_2 measurements

Bloch Equations

• The full Bloch equations (in homogeneous form) are given below
•Three interactions: precession, rf, relaxation

$$\frac{d}{dt}\begin{pmatrix} I_{+1} \\ I_0 \\ I_{-1} \\ I_{eq} \end{pmatrix} = \begin{pmatrix} i\omega - R_2 & i\gamma B_1 & 0 & 0 \\ i\gamma B_1 & -R_1 & i\gamma B_1 & R_1 \\ 0 & i\gamma B_1 & -i\omega - R_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} I_{+1} \\ I_0 \\ I_{-1} \\ I_{eq} \end{pmatrix}$$

Solutions

• Any two interactions - easy
• Full solution is messy because matrix is not Hermitian
• Original solution with Laplace transforms [1-3]
• New solution with more familiar methods [4]

Experimental

• Cyclohexane in $CDCl_3$ – vary T_2 with temperature
• Shigemi tube for good rf homogeneity (810/90 = 92%)
• γB_1 = 500 Hz (180° pulse = 1 ms)
• CPMG under ideal conditions (hard pulses, on resonance) gave same T_2 as Hahn with soft pulse on resonance

Results

• on-resonance: smooth decay
• Off-resonance: added oscillation at frequency offset from resonance (figure 1)
• Tilted effective field

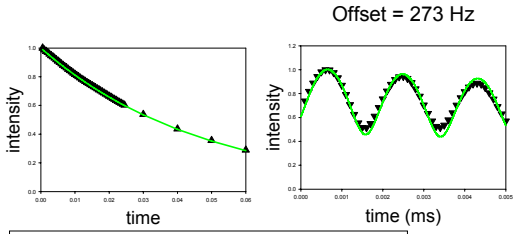

Figure 1
(left) T_2 decay on resonance
(right) T_2 decay 273 Hz off resonance

Data Analysis

• Still a formula for data, but now includes offset
• Simple experiment, more complex data analysis
• Good T_2 out to about $\gamma B_1/2$ (figure 2)
• *e.g.* 10 µs 90 => good to ± 12.5 KHz.

Measured $R2$ vs. Reduced Offset

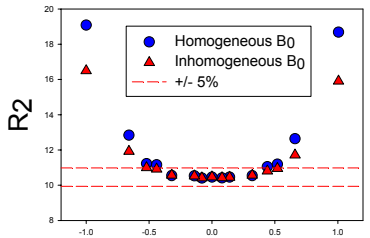

Figure 2
Measured $1/T_2$ as a function of offset

References

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