Teardrop readout gradient waveform design

Ting Ting Ren
Overview

- MRI Background
- Teardrop Model
- Discussion
- Future work
MRI Background: Classical Description of MRI

- **Spins**: MR – relevant nuclei, like $^1$H.
- **Main Field $B_0$**: The magnetic moment vectors tend to align in the direction of $B_0$ to create a net magnetic moment, the nuclear spins exhibit resonance at the Larmor frequency.
- **Radio frequency (RF) field $B_1$**: applied in the xy plane to excite these spins out of equilibrium.
- **Gradient fields $G$**: Phase encoding, frequency encoding
MRI Background: Linear Gradient fields $G$

Square water object:
(a) Given only $B_0$.
(b) Given $B_0$ and linear gradient field.

For 2D imaging, we need $G_x, G_y$. 
MRI Background: Signal Equation and $\kappa$-Space

- Signal Equation

$$s(t) = \int_x \int_y m(x, y) e^{-i2\pi[\kappa_x(t)x + \kappa_y(t)y]} \, dx \, dy$$

Where

$$\kappa_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) \, d\tau$$

$$\kappa_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) \, d\tau$$

$m(x, y)$: the amplitude distribution of excited spins

- The imaging problem becomes one of acquiring the appropriate set of signals $\{s(t)\}$ to enable inversion of signal equation to determine $m(x, y)$. 
MRI Background: Signal Equation and $\kappa$ -Space

- Comparing the signal equation with the 2D Fourier transform of $m(x, y)$,

$$M(\kappa_x, \kappa_y) = \int_{x} \int_{y} m(x, y)e^{-i2\pi[\kappa_x(t)x + \kappa_y(t)y]} \, dx \, dy$$

$$s(t) = M(\kappa_x, \kappa_y)$$

- The total recorded signal $s(t)$ maps directly to a trajectory through Fourier transform space as determined by the time integrals of the applied gradient waveforms $G_x(t)$ and $G_y(t)$.

- In MRI, 2D Fourier transform space is often called “$\kappa$ -Space”, where $\kappa$ represents the spatial frequency variable.

- Proper image formation depends on the appropriate coverage in $\kappa$ -Space.
MRI Background: Spatial Frequency Patterns and 2D Imaging Methods

- Radial Projection Reconstruction
- Standard 2DFT Imaging
- Blipped EPI Imaging
MRI Background: Spatial Frequency Patterns and 2D Imaging Methods

Spiral

Square spiral

Interleaved spiral

Resample
Inverse FT

• Fast Imaging: acquire a greater portion of κ-space per signal readout.
• The gradient system must be able to generate the trajectory.
Teardrop Model: The teardrop $\kappa$-Space trajectory

- The teardrop gradient waveform is continuous family of waveforms, one extreme of which integrates to describe a teardrop shaped $\kappa$-Space trajectory.

- It is designed to follow an interleaved spiral like trajectory leaving the center of $K$-Space, become tangent to a circle at the required resolution, and returning on the mirror image trajectory to the center of $\kappa$-Space.
Teardrop Model: Gradient Waveform

The actual waveform is generated numerically and can be designed interactively to match requested TR and resolution.
Teardrop Model: Advantages of Teardrop

- By using a non-raster trajectory beginning and ending in the center of $\kappa$-Space, a teardrop readout requires neither read nor phase dephase lobes, increasing scan time efficiency.

- By resampling the center of $\kappa$-Space at the beginning of every shot, reconstruction can compensate for the approach to steady state, and the sequence is less sensitive to motion artifacts.
Teardrop Model: Discrete Model

\[
\begin{align*}
& \text{maximize} \quad \| k_{N/2} \| \\
& \text{s.t.} \quad k_0 = 0, \\
& \quad k_N = 0, \\
& \quad g_{-1} = 0, \\
& \quad g_{N+1} = 0, \\
& \quad \| g_i \| \leq G_{\text{max}}, \quad i \in [0, N] \\
& \quad \| g_i - g_{i-1} \| \leq S_{\text{max}}, \quad i \in [0, N+1] \\
& \quad (k_i \cdot g_i)(k_i \cdot g_i)(k_i \cdot k_i) \leq K(k_i^\perp \cdot g_i)(k_i^\perp \cdot g_i) \quad i \in [0, N]. \\
\end{align*}
\]

where \( g_i \in \mathbb{R}^2, \quad i \in [-1, N+1] \)

\[
k_i = \sum_{j=0}^{i} g_j.
\]
Teardrop Model: The spiral constraints

\[ r' = k' |_{azimuthal} = k' \cdot \frac{k}{\sqrt{k \cdot k}} = g \cdot \frac{k}{\sqrt{k \cdot k}} \]

\[ |k'| = k' |_{radial} = k' \cdot \frac{k}{\sqrt{k \cdot k}} = g \cdot \frac{k}{\sqrt{k \cdot k}} \]

\[ r' \leq \alpha \theta' \iff \frac{g \cdot k}{\sqrt{k \cdot k}} \leq \alpha \frac{g \cdot k}{k \cdot k} \]

\[ K = \alpha^2 \]

This constraint is meant to ensure that the trajectory is inside a standard spiral trajectory.
Teardrop Model: Discussion

- Optimal version of teardrop waveform design.

- Can add other constraints, like add velocity compensation constraint to model the effect of flowing velocity on the image.

- Have demonstrated the feasibility of the technique.
Future Work

- Formulate a new teardrop model using a series of LP problems to optimize the waveform.
- Test the linear model by some linear solvers.
- Compare results and quality with the original model.
- Improve the model and apply interior point method to develop a fast and embeddable solver.
- Apply the technique to 3D imaging.