Preface

The “McMaster Optimization Workshop” May 23-24, 2002, held in the ”Information Technology Building” (ITB) of McMaster University, Hamilton, Ontario, Canada. The workshop was hosted by the Department of Computing and Software, McMaster University and its Advanced Optimization Lab.

The purpose of the organizers is to bring together various researchers working on some closely related areas and provide a chance for these leading experts to review the most recent developments in optimization theory, algorithms and software and discuss the challenges in the future. The topics of the workshop cover interior-point methods, general nonlinear and semidefinite optimization. A total of 16 invited speakers will report their recent research results on these topics. We hope all the participants will benefit from these talks whose abstracts are collected in this small volume.

The workshop received generous support from McMaster Faculty of Engineering and MITACS: Mathematics of Information Technology and Complex Systems in the frame of the project “New Interior Point Methods and Software for Convex Conic-Linear Optimization and Their Application to Solve VLSI Circuit Layout Problems”.

May 2002

Tom Luo
Jiming Peng
Tamás Terlaky

Program of the Workshop

May 23, 2002	Section I	Software and Implementation
9:00-9:10	Opening	M. Elbestawi, Dean of Engineering
9:10-9:50	S. Wright	Object-oriented software for quadratic programming
9:50-10:30	E.D. Andersen	Conic optimization in MOSEK: Present and Future
10:30-11:00	Break	
11:00-11:40	J. Sturm	Computational aspects of solving mixed semi-definite and second-order cone optimization problems
11:40-12:20	C. Helmberg	A conic bundle package for LP over symmetric cones
12:20-14:00	Lunch	
	Section II	Nonlinear and Nonsmooth Optimization
14:00-14:40	S. Scholtes	Nonsmooth programs with combinatorial constraints
14:40-15:20	A. Yoshise	A generalization of the homogeneous model for CPs
15:20-15:50	Break	
15:50-16:30	J. Sun	Semismooth homeomorphisms and strong stability of semi-definite and Lorentz CPs
16:30-17:10	F. Glineur	Analyzing conic problems involving Single second-order cone constraint
May, 24, 2002	Section III	Semi-definite Optimization and its Applications
9:00-9:40	M. Kojima	SDP and SOCP relaxations of a class of quadratic optimization problems
9:40-10:20	M. Al-Baali	On the performance of new algorithms for large-scale non-linear least squares
10:20-10:50	Break	
10:50-11:30	F. Rendl	Bundle methods applied to combinatorial optimization problems
11:30-12:10	J. Gondzio	Hamiltonian cycle problem, Markov decision processes and interior point methods
12:10-14:00	Lunch	
	Section IV	Nonlinear Optimization and its Application
14:00-14:40	Y. Yuan	On the regularity of trust region-CG algorithms for nonlinear ill-posed problems
14:40-15:20	J. Zhang	A theoretical analysis on efficiency of some Newton-PCG methods
15:20-15:50	Break	
15:50-16:30	H.G. Bock	Real time optimization and model predictive control of large DAE modelled processes with application to chemical engineering
16:30-17:10	T. Luo	Optimization transceiver design for multi-user communication
17:10-17:20	Closing	

On the performance of new algorithms for large-scale nonlinear least-squares

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Finding a solution to a large-scale nonlinear least-squares optimization problem will be considered. Based on the structure of this problem, certain modification techniques to the limited-memory (L-) BFGS method of Nocedal will be given. Switching between these techniques and that of the Gauss-Newton will be described. A useful feature of these switching L-BFGS/Gauss-Newton algorithms is that they converge quadratically when applied to zero residual problems. Numerical results will be presented to illustrate the performance of these algorithms.

Conic optimization in MOSEK: Present and future

Erling D. Andersen

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Recently, we have added a conic optimizer to the optimization software package MOSEK which solves conic quadratic optimization problems. In this talk we report our experiments with this conic optimizer. We will discuss whether convex quadratically constrained optimization problems should be solved in conic form or in their traditional non-conic form. Finally we will discuss planned improvements to the conic optimizer and possible modelling languages interfaces.

Real time optimization and model predictive control of large DAE modeled processes with application to chemical engineering

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Analyzing conic problems involving a single second-order cone constraint

Francois Glineur

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We consider the standard conic convex problem involving a single second-order cone constraint. No constraint qualification (e.g. Slater) condition is assumed, so that unpleasant features such as non-attainment of the optimum objective value may happen.

We first tackle the associated feasibility problem, which amounts to deciding whether the problem is strictly feasible, weakly feasible, weakly infeasible or strongly infeasible. We show that this can be determined by solving a fixed number of linear equality systems, without requiring any iterative process.

We then outline how this procedure can be modified in order to solve the original optimization problem. We also discuss the point of view of algorithmic complexity and possible generalizations of this procedure.

Hamiltonian cycle problem, Markov decision processes and interior point methods[1]

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The Hamiltonian Cycle Problem (HCP) consists in finding a cycle in a directed graph that enters every node exactly once, or determine that no such cycle exists. A new approach to HCP is discussed. It is built upon:

- embedding the problem in a Markov Decision Process [3];
- reformulating the problem to a non-convex quadratic program [2];
- solving the latter with an interior point method [1].

The well-known example of HCP is the Knight's Tour Problem. It consists in finding a cycle of the Knight through the $k \times k$ chessboard visiting every square exactly once. We have applied our approach to solve a number of such problems with the size of the board reaching 32×32.

Although at the moment our approach is an intellectual exercise rather than the practical method for HCP, we have not lost hope to make it competitive. In this talk we will report on the progress made so far. In particular, we will discuss:

- the use of interior point method for solving non-convex quadratic programs; and
- the use of interior point method within the Branch and Bound procedure.

Both these issues are crucial to make our approach viable.

References

[1] A. Altman and J. Gondzio, Regularized symmetric indefinite systems in interior point methods for linear and quadratic optimization, *Optimization Methods and Software* 11-12 (1999) 275-302.

[2] V. Ejov, J. Filar and J. Gondzio, MDP-based optimization algorithm for Hamiltonian Cycle Problem, in preparation, 2002.

[1] This is a joint work with Vladimir Ejov and Jerzy Filar from the University of South Australia.

[3] J. Filar and D. Krass, Hamiltonian Cycles and Markov Chains, *Mathematics of Operations Research* 19 (1994) 223-237.

A conic bundle package for linear programming over symmetric cones

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Tools for Lagrangian relaxation that produce dual bounds as well as approximate primal solutions are in high demand in combinatorial optimization and stochastic programming. In principle, the spectral bundle approach may be seen as such a tool for conic linear programming over bounded sets. In practice, a software package has to provide more: special support for linear and second order cones, for block structure, and an oracle interface for user designed convex functions. We describe our efforts to offer this in the software package ConicBundle. Our code is designed for minimizing a sum of convex functions where each function is represented by an individual conic bundle. In particular, we present a bundle model for second order cone problems that allows to exploit the special structure like in the semidefinite case. Unfortunately, this model is not as easily generalized to second order cone problems with block structure. We also sketch some possibilities to deal with problems where no apriori bound on the size of a primal solution is known.

SDP and SOCP relaxations of a class of quadratic optimization problems

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SDP (Semidefinite Programming) and lift-and-project LP (Linear Programming) relaxations have been used to compute bounds for objective values of nonconvex QOPs (Quadratic Optimization Problems). The former relaxations are known to be more effective but more expensive than the latter in general. SOCP (Second Order Cone Programming) relaxations serve as a reasonable compromise between the effectiveness of SDP relaxations and the low computational cost of lift-and-project LP relaxations. We present a class of QOPs whose exact optimal values can be computed via SDP and SOCP relaxations.

Optimal transceiver design for multi-user communication

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In this talk, we describe a formulation of the MMSE (Minimum Mean Square Error) transmitter design problem for a multi-user communication system employing either zero-forcing equalizers or MMSE equalizers. Since the natural formulations of this problem turn out to be nonconvex, we develop various alternative formulations using techniques of linear matrix inequalities (LMIs), geometric programming and second order cone programming. For the case of zero-forcing equalizers, we propose an alternating direction method to solve the optimal transmitter design problem and establish its convergence. When restricted to the diagonal designs, we further simplify the formulation to a linearly constrained entropy maximization problem which can be efficiently solved. In the case where MMSE equalizers are used at the receivers, we formulate the optimal transmitter design problem as a semidefinite program (SDP) which can be solved using the highly efficient interior point methods. When the channel matrices are diagonal (as in OFDM systems), we show that the optimal MMSE transmitters can be obtained by subcarrier allocation and optimal power loading to each subcarrier for all the users. Moreover, the optimal subcarrier allocation and power-loading can be computed fairly simply by the relative ratios of the path gains corresponding to all subcarriers.

Semismooth homeomorphisms and strong stability of semidefinite and lorentz complementarity problems

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Based on an inverse function theorem for a system of semismooth equations, we establish several necessary and sufficient conditions for an isolated solution of a complementarity problem defined on the cone of symmetric positive semidefinite matrices to be strongly regular/stable. A similar result is also derived for a complementarity problem defined on the Lorentz cone. The analysis relies on new interpretations of the directional derivatives of the projector onto the semidefinite and the Lorentz cone, which are of independent interest.

Bundle methods applied to combinatorial optimization problems

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Several hard combinatorial optimization problems, such as Max-Cut or Quadratic Assignment can be approximated nicely by Semidefinite Programs (SDP). These relaxations can be further improved by including combinatorial cutting planes, resulting in huge SDPs. We use the bundle method from nonsmooth optimization to tackle these SDPs. The idea consists in identifying a basic model, and taking the Lagrange dual of the remaining constraints. We discuss some practical issues like finding good primal solutions, and identifying important cutting planes. Finally, we present computational results of this approach applied to the max-cut relaxation given by the basic SDP model intersected with the metric polytope, and SDP relaxations of the Quadratic Assignment Problem.

Nonsmooth programs with combinatorial constraints

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Presently, the most popular methods for nonsmooth optimization are variants of the bundle method for convex optimization. These procedures are appealing since they only require the user to compute function values and subgradients. The methods were originally designed for convex problems but they are well-defined for general Lipschitz functions if a suitable subgradient concept is used. In the latter case, however, they exhibit rather weak convergence properties - not surprisingly since they are based on convex models. We study a special, albeit fairly broad class of nonsmooth problems which arise, e.g., through the use of ”max”, ”min”, or ”if” statements in function evaluations and propose a general approach, reminiscent of SQP, which exhibits more favorable convergence properties.

Computational aspects of solving mixed semidefinite and second order cone optimization problems

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This talk presents an accessible survey of solving mixed semidefinite and second order cone optimization problems using the primal-dual interior point method. Using certain Jordan algebra concepts, the role of acronyms like AHO, NT, HKM in the the search direction subproblem is clarified. Details of one or more implementational topics will also be revealed.

Object-oriented software for quadratic programming[2]

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We describe the object-oriented software package OOQP for solving convex quadratic programming problems (QP). The primal-dual interior point algorithms supplied by OOQP are implemented in a way that is largely independent of the problem structure. Users may exploit problem structure by supplying linear algebra, problem data, and variable classes that are customized to their particular applications. The OOQP distribution contains default implementations that solve several important QP problem types, including general sparse and dense QPs, bound-constrained QPs, and QPs arising from support vector machines and Huber regression. The implementations supplied with the OOQP distribution are based on such well known linear algebra packages as MA27/57, LAPACK, and PETSc.

[2]This talk represents joint work with Mike Gertz.

On the regularity of trust region-CG algorithms for nonlinear ill-posed problems

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In this paper we consider the regularity of the trust region-cg algorithm, which has been used for image reconstruction problems. This method is different from the traditional regularization method, because it does not need to introduce a penalty term, called the stable functional, hence the determining of the so-called regularization parameter is avoided. Theoretical analysis of the trust region-cg method is presented, convergency and regularity of the trust region algorithm are proved, numerical test is also given.

A generalization of the homogeneous model for complementarity problems

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It is well-known that the homogeneous algorithm enjoys many desirable properties when we apply it to linear programs or monotone complementarity problems. However, the algorithm is quite different from other interior point algorithms: the monotonicity of the problem plays an essential role in the analysis and it seems difficult to extend the algorithm to more generalized problems as it is. In this talk, we will provide a generalized homogeneous model which can be applied to P_* complementarity problems and discuss the induced mapping, the existence of the trajectory, the solution set, and so on.

A theoretical analysis on efficiency of some Newton-PCG methods [3]

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In this paper, we study the efficiency issue of inexact Newton-type methods for smooth unconstrained optimization problems under standard assumptions from theoretical point of view by discussing a concrete Newton-PCG algorithm. In order to compare the algorithm with Newton's method, a ratio between the measures of their approximate efficiencies is investigated. Under mild conditions, it is shown that first, this ratio is larger than 1, indicating that the Newton-PCG algorithm is more efficient than Newton's method, and second, this ratio increases when the dimension n of the problem increases and tends to infinity at least at a rate $\ln n/\ln 2$ when $n \to \infty$, which implies that *in theory* the Newton-PCG algorithm is much more efficient for middle and large-scale problems. These theoretical results are also supported by our preliminary numerical experiments.

[3]This work was supported by Hong Kong Competitive Earmarked Research Grant CityU 1066/00P from the Hong Kong University Grant Council and the National Science Foundation of China Grant No.10071094.

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