Assignment 2

Due. February 12, Wednesday, 9:00.

1. Evaluate the infinite sum

$$\Phi(x) = \sum_{k=1}^{\infty} \frac{1}{k(k+x)} \quad \text{for } x = 0, 0.1, 0.2, ..., , 1.0,$$

with an error less than 0.5×10^{-8}.

Note: This requires both human analysis and computer power, and neither is likely to succeed without the other. Above all, do not waste computer budget trying to sum the series by brute force.

Hint: Use the fact that

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

to prove that $\Phi(1) = 1$. Then express $\Phi(x) - \Phi(1)$ as an infinite series which converges faster than the one defining $\Phi(x)$. You will have to repeat this trick before you get a series for computing $\Phi(x)$ that converges fast enough. To determine the convergence of the new series, you may use the identity:

$$\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}.$$

The series

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6},$$

which eqals $\Phi(0)$, can be used for testing.

2. Write two Matlab/Octave functions:

```
[d, s, rcond] = tspdChol(d, s)
b = tspdSolve(d,s,b)
```

Suppose a tridiagonal symmetric and positive definite matrix is given in the form of two vectors: the main diagonal d, and the subdiagonal s, the first function `tspdChol` performs the Cholesky factorization and estimates the condition number of the matrix. In the output, d is the main diagonal of the Cholesky factor L, s the lower subdiagonal of L, and *rcond* the reciprocal of the estimated condition number. Also, this function should detect singularity and non positive definiteness. The second function `tspdSolve` takes the outputs from `tspdChol` as inputs and solves the tridiagonal symmetric and positive definite system with the right-side vector b. On return, the solution is stored in b. In your program, you should operate on the vectors. Do not form matrices in the functions. Actually, you don't even need auxiliary vectors. For submission, along with the two functions `tspdChol.m` and `tspdSolve.m`, explain the tests carried out. The functions should be well documented following the style of the sample programs.

3. The following figures from the Census Bureau give the population of the United States:

Year	Population
1900	75,994,575
1910	91,972,266
1920	105,710,620
1930	122,775,046
1940	131,669,275
1950	150,697,361
1960	179,323,175
1970	203,235,298

Since there are eight points, there is a unique polynomial of degree 7 which interpolates the data. However, some of the ways of representing this polynomial are computationally more satisfactory than others. Here are four possibilities, each with t ranging over the interval $1900 \le t \le 1970$:

$$\sum_{j=0}^{7} a_j t^j,$$
$$\sum_{j=0}^{7} b_j (t-1900)^j,$$
$$\sum_{j=0}^{7} c_j (t-1935)^j,$$
$$\sum_{j=0}^{7} d_j \left(\frac{t-1935}{35}\right)^j.$$

In each case, the coefficients are found by solving an 8-by-8 Vandermond system, but the matrices of various systems are quite different. Set up each of the four matrices, then use `decomp` and `solve` to estimate its condition number and find the coefficients by solving the system. Check each of the representations to see how well it reproduces the original data.