

CAS 708 Scientific Computation

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Course web page

My home page:

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- announcements
- my address, office hours
- lecture notes, no text book, references for each chapter
- assignments

About this course

Computer methods for solving problems scientific and engineering applications.

Evaluation scheme

- Three written/programming (MATLAB, Octave) assignments 30%
- Midterm (closed book, 1 hr) 20%
- Final (open book, 2 hrs) 50%
- Missing assignments may be combined with later assignments (not exams)
- A deferred midterm will be offered

Example

The propagation of sound under water.

The speed of sound in ocean water depends on pressure, temperature, and salinity, all of which vary with depth in fairly complicated ways.

Problem:

A sound source at a depth of 2000 feet transmits to a receiver 24 miles away at a depth of 3000 feet. At what angle(s) the ray leaving at the source will reach the receiver?

Example

Since the sound speed varies with depth, sound rays will travel in curved paths. The effect is continuous version of the familiar refraction of light waves caused by the air-water interface in a fish bowl. The basic equation is a continuous version of Snell's law:

$$\frac{\cos \theta}{c(z)} = \text{constant.}$$

where z denotes the depth of a particular ray at distance x , (z is a function of x), $c(z)$ is the speed of sound at depth z , and $\theta = \theta(x)$ denote the angle between a horizontal line and the tangent to the ray at x , i.e.,

$$\tan \theta = \frac{dz}{dx}.$$

Example

These two equations together yield a second order ordinary differential equation

$$\frac{d^2z}{dx^2} = -\frac{c'(z)}{A^2c(z)^3},$$

where A is the constant occurring in Snell's law.

This second order differential equation is then transformed into a system of two first order equations.

Example

Problem:

Solving a system of two first order ordinary differential equations with initial conditions:

the depth of the source $z(0)$

the ray angle at the source

Requires $c(z)$, speed at depth z used in the solver.

Example

Measure the speed of sound at various depths:

depth z	0	500	1000	...	11000	12000
speed $c(z)$	5042	4995	4948	...	4973	4991

The ODE solver requires $c(z)$ at depths other than those depths.

Example

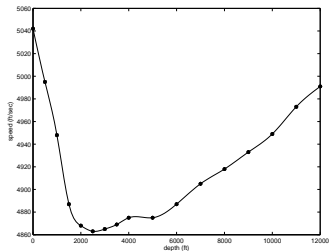
Problems:

Construct a function that interpolates the measurements, for example, $c(0) = 5042$, $c(500) = 4995$. But it can be evaluated at any value of z between 0 and 12,000, such as $z(780)$.

Example

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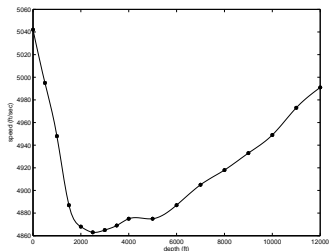
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Constructing an interpolation function requires solving a system of linear equations.

Example

For a given initial angle at source, by solving the differential equation (involving the evaluation of an interpolation function), we obtain the depth of the ray 24 miles away. We want to find the initial angle(s) so that the depth is 3000 feet.

Problem:

Solving a nonlinear equation $f(\theta_0) - 3000 = 0$, where f represents the above quite complicated process and θ_0 the initial angle.

Text book formulas vs computer methods

Solving a quadratic equation

$$ax^2 + bx + c = 0, \quad a \neq 0$$

Text book formula:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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Computer method:

$$x_1 = \frac{2c}{-b - \text{sign}(b)\sqrt{b^2 - 4ac}}, \quad x_2 = \frac{c}{ax_1}$$

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Mathematically equivalent, but more efficient, more accurate, and more robust.

Real vs floating-point

Math formulas use real variables, whereas computer programs use floating-point variables (float, double).

Floating-point numbers (finite precision) are approximations of real numbers (continuous).

We will first discuss the differences between real numbers and floating-point numbers. The implications of finite precision.

Let's work together.

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Good luck!