Assignment 1

Due. Sept. 24, Monday, 13:30.

1. Give the IEEE single precision binary representation of each of the following decimal numbers:
   $+2$
   -33
   $+1.3$
   -0.4

2. How many IEEE single precision numbers x satisfy $1.0 \leq x < 2.0$?

3. Consider the following program:

```
h = 1.0/2.0;
s = 2.0/3.0 - h;
t = 3.0/5.0 - h;
d = (s + s + s) - h;
n = (t + t + t + t + t) - h;
q = n/d;
```

   The variable q can take on different values depending on the floating-point system used by the computer.

   (a) Figure out the value of q, if the program is run in MATLAB/Octave (double precision). Explain the result.

   (b) Figure out the value of q, if the program is run in single precision. Explain your result.

   (c) Figure out the value of q, if the program is run on a hypothetical machine with $\beta = 10$, $t = 4$, $e_{\min} = -48$, and $e_{\max} = 49$.

4. In 250 B.C.E. the Greek mathematician Archimedes estimated the number π as follows. He looked at a circle with diameter 1, and hence circumference π. Inside the circle he inscribed a square. The perimeter of the square is smaller than the circumference of the circle, and so it is a lower bound for π. Archimedes then considered an inscribed octagon, 16-gon, etc., each time doubling the number of sides of the inscribed polygon, and producing ever better estimates for π. Using 96-sided inscribed and circumscribed polygons, he was able to show that $223/71 < \pi < 22/7$. There is a recursive formula for these estimates. Let p_n be the perimeter of the inscribed polygon with 2^n sides. Then $p_2 = 2\sqrt{2}$. In general,

$$p_{n+1} = 2^n \sqrt{2(1 - \sqrt{1 - (p_n/2^n)^2})}$$

   Compute p_n for $n = 3, 4, ..., 60$. Try to explain your results.

   Kahan suggested a revision:

$$p_{n+1} = 2^n \sqrt{r_{n+1}}$$

where r_{n+1} can be computed iteratively

$$r_{n+1} = \frac{r_n}{2 + \sqrt{4 - r_n}} \qquad r_3 = \frac{2}{2 + \sqrt{2}}.$$

Use this revision to calculate r_n and p_n for $n = 3, 4, ..., 60$. Try to explain your results.