Assignment 1


1. Give the IEEE single precision binary representation of each of the following decimal numbers:
   +2
   −33
   +1.3
   −0.4

2. How many IEEE single precision numbers $x$ satisfy $1.0 \leq x < 2.0$?

3. Consider the following program:

   ```
   h = 1.0/2.0;
   s = 2.0/3.0 - h;
   t = 3.0/5.0 - h;
   d = (s + s + s) - h;
   n = (t + t + t + t + t) - h;
   q = n/d;
   ```

   The variable $q$ can take on different values depending on the floating-point system used by the computer.

   (a) Figure out the value of $q$, if the program is run in MATLAB/Octave (double precision). Explain the result.

   (b) Figure out the value of $q$, if the program is run in single precision. Explain your result.

   (c) Figure out the value of $q$, if the program is run on a hypothetical machine with $\beta = 10$, $t = 4$, $e_{\min} = -48$, and $e_{\max} = 49$.

4. In 250 B.C.E. the Greek mathematician Archimedes estimated the number $\pi$ as follows. He looked at a circle with diameter 1, and hence circumference $\pi$. Inside the circle he inscribed a square. The perimeter of the square is smaller than the circumference of the circle, and so it is a lower bound for $\pi$. Archimedes then considered an inscribed octagon, 16-gon, etc., each time doubling the number of sides of the inscribed polygon, and producing ever better estimates for $\pi$. Using 96-sided inscribed and circumscribed polygons, he was able to show that $\frac{223}{71} < \pi < \frac{22}{7}$. There is a recursive formula for these estimates. Let $p_n$ be the perimeter of the inscribed polygon with $2^n$ sides. Then $p_2 = 2\sqrt{2}$. In general,

   $$p_{n+1} = 2^n \sqrt{2(1 - \sqrt{1 - (p_n/2^n)^2})}$$

   Compute $p_n$ for $n = 3, 4, ..., 60$. Try to explain your results.

   Kahan suggested a revision:

   $$p_{n+1} = 2^n \sqrt{r_{n+1}}$$
where $r_{n+1}$ can be computed iteratively
\[
    r_{n+1} = \frac{r_n}{2 + \sqrt{4 - r_n}} \quad r_3 = \frac{2}{2 + \sqrt{2}}.
\]

Use this revision to calculate $r_n$ and $p_n$ for $n = 3, 4, ..., 60$. Try to explain your results.