# Solution for Assignment 3

1. (12 marks) The following figures from the Census Bureau give the population of the United States:

| Year | Population |
|---|---|
| 1900 | 75,994,575 |
| 1910 | 91,972,266 |
| 1920 | 105,710,620 |
| 1930 | 122,775,046 |
| 1940 | 131,669,275 |
| 1950 | 150,697,361 |
| 1960 | 179,323,175 |
| 1970 | 203,235,298 |

- Since there are eight points, there is a unique polynomial of degree 7 which interpolates the data. However, some of the ways of representing this polynomial are computationally more satisfactory than others. Here are four possibilities, each with $t$ ranging over the interval $1900 \le t \le 1970$:

$$\sum_{j=0}^{7} a_j t^j,$$

$$\sum_{j=0}^{7} b_j (t-1900)^j,$$

$$\sum_{j=0}^{7} c_j (t-1935)^j,$$

$$\sum_{j=0}^{7} d_j \left(\frac{t-1935}{35}\right)^j.$$

  In each case, the coefficients are found by solving an 8-by-8 Vandermond system, but the matrices of various systems are quite different. Set up each of the four matrices, and find the estimate of its condition using Matlab/Octave function `cond()`. Then use Matlab/Octave operator "\" to find the coefficients. Check each of the representations to see how well it reproduces the original data.

- Interpolate the data by a 7th-degree polynomial, using the best conditioned representation found above, and by the natural cubic spline using `ncspline.m`. Graph the resulting functions at one-year intervals over the period from 1900 to 1980. Find the 1980 census data. Which approach gives more accurate prediction?

**Solution**: The condition numbers:

| | model a | model b | model c | model d |
|---|---|---|---|---|
| cond | 1.212e+32 | 1.785e+13 | 7.891e+10 | 5.354e+2 |

Relative errors of the reproduced data by evaluating the polynomials using the Horner's rule:

| | model a | model b | model c | model d |
|---|---|---|---|---|
| relative error | 4.0−3 | 4.7e−14 | 2.3e−16 | 4.2e−16 |

Predictions for 1980:

| | model d | spline | real |
|---|---|---|---|
| prediction | 402.33 million | 227.15 million | 226.44 million |

2. (12 marks) Modify `QUADR` so that it returns `fcnt` as the total number of function evaluations and `minl` as the length of the smallest panel which it uses. Then write a MATLAB/Octave program `QUADS` replacing the rectangle rule with the Simpson's rule. Run both programs on a fairly hard problem such as $f(x) = \sqrt{x}$. Compare the numbers of function evaluations and the lengths of the smallest panels.

   **Solution**: See `QUADRm.m`, `quadrrm.m`, `QUADS.m`, and `quadsr.m`. For example, $f(x) = \sqrt{x}$ and $\text{tol} = 0.0001$,

   | | function evaluation count | min interval length |
   |---|---|---|
   | Rectangle | 91 | $2^{-10}$ |
   | Simpson's | 37 | $2^{-9}$ |