Assignment 4

Due. Dec. 3, Monday, 11:30.

1. (12 marks) A famous problem of nonlinear mechanics is known as the *inverted pendulum.* The pendulum is a stiff bar of length L which is supported at one end by a frictionless pin. The support pin is given a rapid up-and-down motion s by means of an electric motor:

$$s = A \sin \omega.$$

A simple application of Newton's second law of motion yields the quation of motion

$$\theta'' = \frac{3}{2L}(g - A\omega^2 \sin(\omega t)) \sin \theta,$$

where g is the acceleration due to gravity. For small values of θ, $\sin\theta \approx \theta$, and this question becomes the well-known Mathieu equation, which is known to be stable for certain values of A and ω and initial values. For $A = 0$, we have the familiar pendulum equation

$$\theta'' = \frac{3g}{2L} \sin \theta,$$

which can be linearized for values of θ near 0.

The most interesting aspect of this problem is that there are regions in which the equation of motion is stable for initial values corresponding to an inverted configuration, and they have been physically realized. Write a MATLAB/Octave program to compute the motion $\theta(t)$ for various values of L, A, ω and initial values $\theta(0)$ and $\theta''(0)$. Debug your program using the case $L = 10$ in., $A = 0$ in., $\omega = 0$ rad/sec, $\theta(0) = 0.1$ rad, and $\theta'(0) = 0$ rad/sec, where g is taken to be 386.09 in./sec^2. Compare your computed solution with the analytical solution of the linearized version. Print θ and θ' for two or three oscillations of the pendulum.

When your program appears to be working satisfactorily, try the more interesting cases:

L	A	ω	$\theta(0)$	$\theta'(0)$
10	0.5	5.3	3.1	0
10	10	100	3.1	0
10	10	100	0.1	0
10	2	100	0.1	0
10	0.5	200	0.05	0