COSC6397 Homework Assignment 1 (Larger-scale Fading)

Solution

Problem 1: Suppose a transmitter produces 30W of power.

a. Express the transmit power in units of dBm and dBW.
b. If the transmitter's power is applied to a unity gain antenna with a 900-MHz carrier frequency, what is the received power in dBm at a free space distance of 100 m?
c. Repeat (b) for a distance of 10km.
d. Repeat (b) (c) under ground reflected model with the height of the transmitter and receiver being 30m and 1m respectively.

Solution:
(a) 30W = 14.77dB = 44.77dBm
(b)(c) $f = 900MHz$, $\lambda = c/f = 3e8/9e8 = 0.33$. From the Friis free space equation $P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$, at distance 100m, $P_r = -26.76dBm$. At distance 10km, $P_r = -66.76dBm$. (in this problem, we assume the far-field distance $d_0 < 100m$. $L = 1$.)
(d)

- Answer 1: Since $d_c = (4\pi h_t h_r)/\lambda = 1142 > 100m$, the free space model is used. $P_r = -26.76dBm$.
- Answer 2: Under ground reflected model, $P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$, $P_r = 30 * 30^2/100^4 = -35.69dBW$ for distance 10m.

For d = 10km, $P_r = -115.69dBW$.

Problem 2: Prove that in the two-ray ground reflected model, $\Delta = d'' - d' \approx 2h_t h_r/d$. Show when this holds as a good approximation.
Solution:

$$\Delta = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2} \quad (1)$$

$$= \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t + h_r)^2 + d^2 - 4h_t h_r}. \quad (2)$$

Let $x = \sqrt{(h_t + h_r)^2 + d^2}$, then

$$\Delta = x(1 - \sqrt{1 - 4h_t h_r/x^2}) = x(1 - 1 + 2h_t h_r/x^2 - o(4h_t h_r/x^2)) \approx 2h_t h_r/x. \quad (3)$$

The second equality is due to Taylor series $(1 + x)^a = 1 + ax + a(a - 1)x^2 + ...$ and o(x) represents a high order polynomial of x (with exponent larger than 1). The last approximation holds iff (if and only if) $4h_t h_r/x^2 << 1$.

Lastly, when $d >> h_t + h_r$, $x \approx d$. Therefore, from Eq. (3), we have $\Delta \approx 2h_t h_x/d^2$.

Problem 3: Consider seven-cell frequency reuse. Cell B1 is the desired cell and B2 is a co-channel cell as shown in Figure 1(a). For a mobile located in cell B1, find the minimum cell radius R to give a forward link C/I (carrier to interference) ratio of at least 18 dB at least 99% of the time. Assume the following:

Co-channel interference is due to base B2 only.
Carrier frequency, $f_c = 890$MHz.
Reference distance, $d_0 = 1km$ (assume free space propagation from the transmitter to d_0).
Assume omnidirectional antenna for both transmitter and receiver, where $G_{base} = 6$dBi and $G_{mobile} = 3$dBi.
Transmitter power, $P_t = 10W$ (assume equal power for all base stations).
$PL(dB)$ between the mobile and base B1 is given as,

$$\overline{PL}(dB) = \overline{PL}(d_0) + 10(2.5)log(\frac{d_1}{d_0}) - X_\sigma, \sigma = 0dB. \quad (4)$$

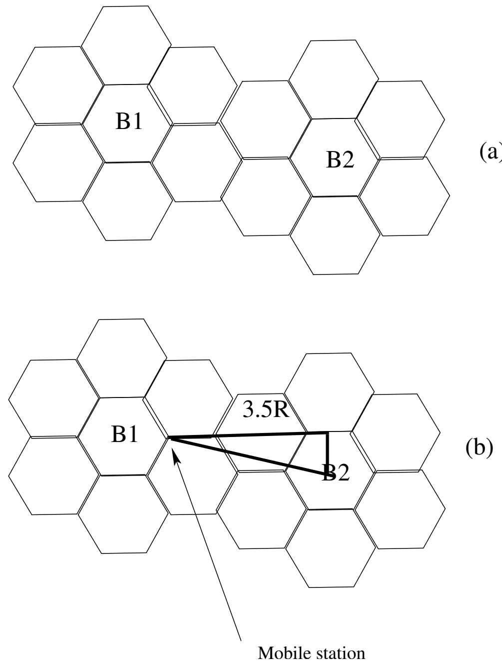

Fig. 1. (a) Seven-cell reuse structure; (b) co-channel interference geometry between B1 and B2

$PL(dB)$ between the mobile and base B2 is given as

$$\overline{PL}(dB) = \overline{PL}(d_0) + 10(4.0)log(\frac{d_2}{d_0}) - X_\sigma,\ \sigma = 7dB. \quad (5)$$

Cell boundaries are shown in Figure 1(b).

Solution:

Consider when the mobile is at the boudary of cell B1 as indicated by Figure 1(a).

The received signal from B1 can be expressed as,

$$\begin{aligned} P_1[dBm] &= Pt[dBm] - PL(d)[dB] \\ &= Pt[dBm] - \overline{PL}(d_0) - 10log((\frac{R}{d_0})^{2.5}) - X_{\sigma_1} \end{aligned} \quad (6)$$

Similarly, the interference level from B2 under the assumption of equal power is,

$$\begin{aligned} P_2[dBm] &= Pt[dBm] - PL(d)[dB] \\ &= Pt[dBm] - \overline{PL}(d_0) - 10log((\frac{3.58R}{d_0})^{4}) - X_{\sigma_2} \end{aligned} \quad (7)$$

Therefore, the C/I ratio can be expressed as,

$$C/I = P_1 - P_2 = -10log((\frac{R}{d_0})^{2.5}) - X_{\sigma_1} + 10log((\frac{3.58R}{d_0})^{4}) + X_{\sigma_2} \quad (8)$$

C/I follows log-normal Gaussian distribution with mean $\mu = 10log((\frac{3.58R}{d_0})^4) - 10log((\frac{R}{d_0})^{2.5}) = 15logR - 22.84dB$, and $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 7dB$ (since the sum of two independent Gaussian distribution (μ_x, σ_x^2) and (μ_y, σ_y^2) follows Gaussian distribution $(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2)$).

To have C/I ratio of at least 18dB at least 99% of the time, the follow condition holds,

$$\frac{18 - \mu}{\sigma} = -\sqrt{2}erf^{-1}(0.49 * 2) \quad (9)$$

where erf is the error function for Gaussian distribution (0, 1) defined as $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x exp(-x^2)dt$. By plugging the expression for μ and σ, we have $R = 6.43Km$.

Problem 4: When A pair of nodes A and B are sending packets to node C using IEEE 802.11 DCF. All nodes are within transmission and carrier sensing range with one another. Both nodes A and B have many packets pending for node C. Show on a timing diagram the sequence of events that occurs until each of nodes A and B has received ACK for their first packet sent to C, assuming that they pick their successive back-off intervals as follows:
Node A: 3, 4, 8, 4, 2
Node B: 7, 6, 5, 15, 17

Assume that the propagation delay is negligible, and that the two nodes choose their initial back-off exactly at time t0, and that at time t0 channel changes status from busy to idle. In your timing diagram, show one time-line each for hosts A, B and C (Fig. 2). In the time-line, show the various packets sent by the hosts, and back-off slots counted by the hosts and inter-frame spacing. Also, if a packet transmission results in a collision, indicate that as well. *No* RTS/CTS is used prior to Data and ACK, and that in the absence of a collision, all transmissions are received reliably.

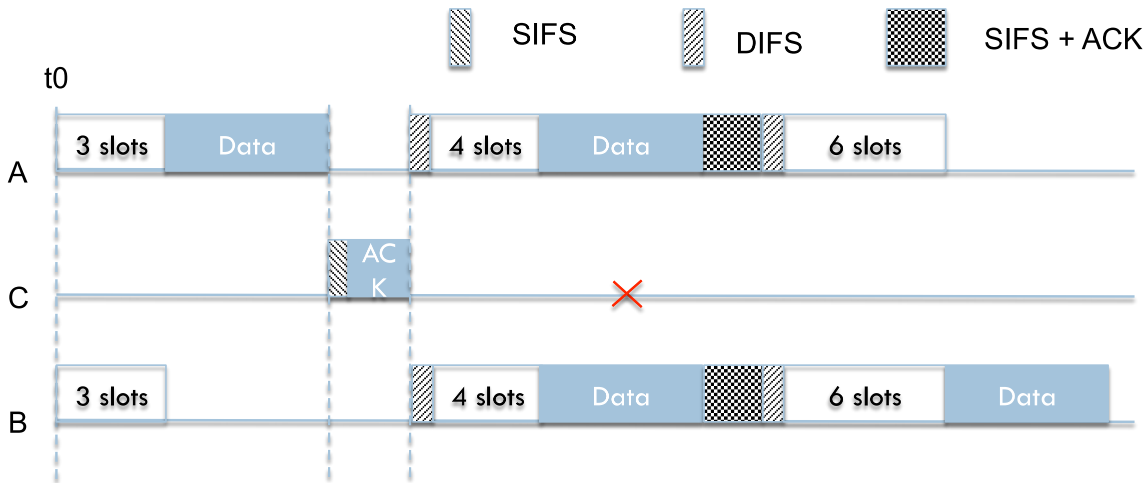

Fig. 2. Time-line for host A, B, C