**COSC6397 Homework Assignment 1 (Larger-scale Fading)**

**Solution**

**Problem 1.** Suppose a transmitter produces 50W of power.

*a.* Express the transmit power in units of dBm and dBW.

*b.* If the transmitter’s power is applied to a unity gain antenna with a 900-MHz carrier frequency, what is the received power in dBm at a free space distance of 100 m?

*c.* Repeat (b) for a distance of 10km.

*d.* Repeat (b) (c) under ground reflected model with the height of the transmitter and receiver being 30m and 1m respectively.

**Solution:**

(a) $50 \text{W} = 16.99 \text{dBW} = 46.99 \text{dBm}$

(b) $f = 900 \text{MHz}$, $\lambda = \frac{c}{f} = \frac{3e8}{9e8} = 0.33$. From the Friis free space equation $P_r(d) = \frac{P_t G_t G_r h_t^2 h_r^2}{(4\pi) d L}$, at distance 100m, $P_r = -24.62 \text{dBm}$. At distance 10km, $P_r = -64.52 \text{dBm}$. (in this problem, we assume the far-field distance $d_0 < 100m$. $L = 1$.)

(d) Under ground reflected model, $P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d}$, $P_r = 50 \times 30^2 / 100^4 = -33.47 \text{dBW}$ for distance 10m. For $d = 10km$, $P_r = -113.47 \text{dBW}$.

**Problem 2.** Prove that in the two-ray ground reflected model, $\Delta = d'' - d' \approx 2h_t h_r / d$. Show when this holds as a good approximation.

**Solution:**

$$\Delta = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2} = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t + h_r)^2 + d^2 - 4h_t h_r}.$$  \hspace{1cm} (1)

Let $x = \sqrt{(h_t + h_r)^2 + d^2}$, then

$$\Delta = x(1 - \sqrt{1 - 4h_t h_r / x^2}) = x(1 - 1 + 2h_t h_r / x^2 - o(4h_t h_r / x^2)) \approx 2h_t h_r / x.$$  \hspace{1cm} (3)

The second equality is due to Taylor series $(1 + x)^a = 1 + ax + a(a-1)x^2 + ...$ and $o(x)$ represents a high order polynomial of $x$ (with exponent larger than 1). The last approximation holds iff (if and only if) $4h_t h_r / x^2 << 1$.

Lastly, when $d >> h_t + h_r$, $x \approx d$. Therefore, from Eq. (3), we have $\Delta \approx 2h_t h_r / d^2$.

**Problem 3.** Consider seven-cell frequency reuse. Cell B1 is the desired cell and B2 is a co-channel cell as shown in Figure 1(a). For a mobile located in cell B1, find the minimum cell radius R to give a forward link C/I (carrier to interference) ratio of at least 18 dB at least 99% of the time. Assume the following:

- Co-channel interference is due to base B2 only.
- Carrier frequency, $f_c = 890 \text{MHz}$.
- Reference distance, $d_0 = 1 \text{km}$ (assume free space propagation from the transmitter to $d_0$).
- Assume omnidirectional antenna for both transmitter and receiver, where $G_{\text{base}} = 6 \text{dBi}$ and $G_{\text{mobile}} = 3 \text{dBi}$.
- Transmitter power, $P_t = 10 \text{W}$ (assume equal power for all base stations).

**PL(dB)** between the mobile and base B1 is given as,

$$PL(dB) = PL(d_0) + 10(2.5)log(\frac{d_1}{d_0}) - X_\sigma, \sigma = 0 \text{dB}.$$  \hspace{1cm} (4)

**PL(dB)** between the mobile and base B2 is given as

$$PL(dB) = PL(d_0) + 10(4.0)log(\frac{d_2}{d_0}) - X_\sigma, \sigma = 7 \text{dB}.$$  \hspace{1cm} (5)
Cell boundaries are shown in Figure 1(b).

**Solution:**
Consider when the mobile is at the boudary of cell B1 as indicated by Figure 1(a).

The received signal from B1 can be expressed as,

\[ P_1[dBm] = Pt[dBm] - PL(d)[dB] = Pt[dBm] - PL(d_0) - 10\log\left(\frac{R}{d_0}^{2.5}\right) - X_{\sigma_1} \]  

(6)

Similarly, the interference level from B2 under the assumption of equal power is,

\[ P_2[dBm] = Pt[dBm] - PL(d)[dB] = Pt[dBm] - PL(d_0) - 10\log\left(\frac{3.58R}{d_0}^{4}\right) - X_{\sigma_2} \]  

(7)

Therefore, the C/I ratio can be expressed as,

\[ \frac{C}{I} = P_1 - P_2 = -10\log\left(\frac{R}{d_0}^{2.5}\right) - X_{\sigma_1} + 10\log\left(\frac{3.58R}{d_0}^{4}\right) + X_{\sigma_2} \]  

(8)

C/I follows log-normal Gaussian distribution with mean \( \mu = 10\log\left(\frac{3.58R}{d_0}^{4}\right) - 10\log\left(\frac{R}{d_0}^{2.5}\right) = 15\log R - 22.84dB \), and \( \sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 7dB \) (since the sum of two independent Gaussian distribution \( (\mu_x, \sigma_x^2) \) and \( (\mu_y, \sigma_y^2) \) follows Gaussian distribution \( (\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2) \)).

To have C/I ratio of at least 18dB at least 99% of the time, the follow condition holds,

\[ \frac{18 - \mu}{\sigma} = \sqrt{2}\operatorname{erf}^{-1}(0.01 \times 2) \]  

(9)

where \( \operatorname{erf} \) is the error function for Gaussian distribution \((0, 1)\) defined as \( \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-x^2) \, dx \). By plugging the expression for \( \mu \) and \( \sigma \), we have \( R = 6.43Km \).