

# Concurrent Processes

## SE 3BB4

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- Concurrent process is a **composition** of sequential processes.
- *Hidden assumption: Concurrent systems can be decomposed into sequential systems.*

- ① Process (sequential): A sequence of action



- ② Model of a process: Finite state machine



- ③ A possible implementation of processes: Threads in Java.

The approach 1,2,3 is not the only one, but we will concentrate on it in this course.

Concepts: Processes - units of sequential execution

Models: Finite State Processes (FSP)

To model processes as sequences of actions

Labelled Transition Systems (LTS)

To analyze, display and animate behaviour

Practice: Java threads

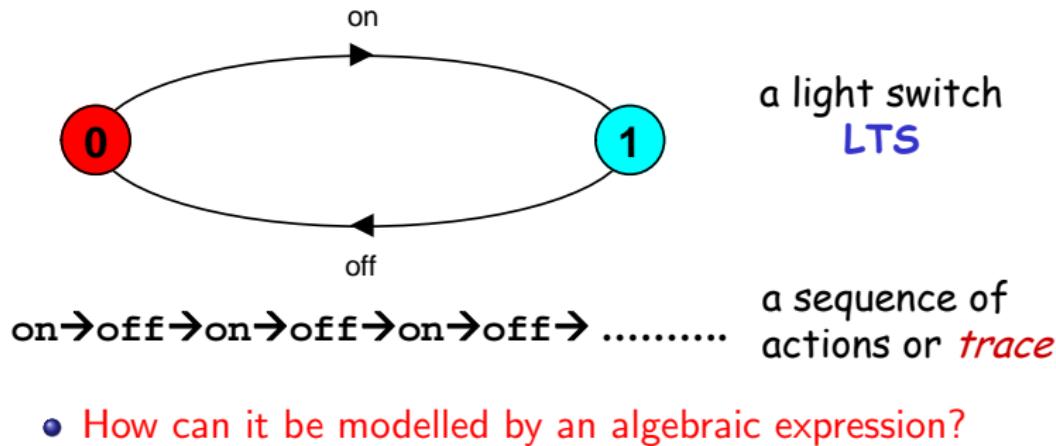
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- LTS - graphical form
- FSP - algebraic form

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- Tool LTSA takes FSP and analyses them.
- Different names for the same concepts:  
LTS - automata, state machines  
FSP - CSP (Communicating Sequential Processes), Processes in Process Algebras

- A process is the execution of a sequential program. It is modeled as a finite state machine which transits from state to state by executing a sequence of atomic actions.



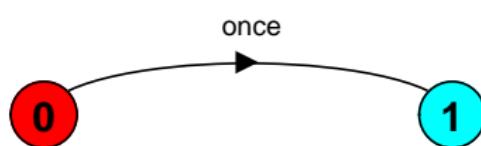
- How can it be modelled by an algebraic expression?

- If  $x$  is an action and  $P$  is a process then

$$(x \rightarrow P)$$

describes a process that initially engages in the action  $x$  and then behaves exactly as described by  $P$ .

**ONESHOT** = **(once** -> **STOP**) .



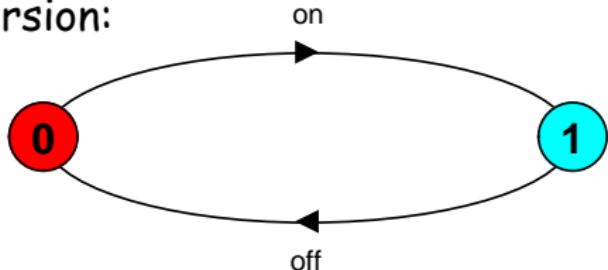
**ONESHOT state machine**

**(terminating process)**

- Convention: actions begin with lowercase letters while PROCESSES begin with uppercase letters

Repetitive behaviour uses recursion:

```
SWITCH = OFF,  
OFF    = (on -> ON),  
ON     = (off-> OFF).
```



Substituting to get a more succinct definition:

```
SWITCH = OFF,  
OFF    = (on ->(off->OFF)).
```

And again:

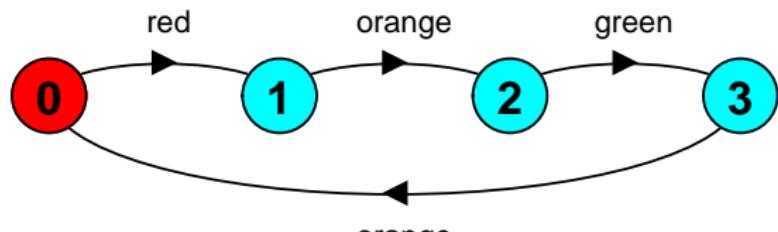
```
SWITCH = (on->off->SWITCH).
```

# FSP model of a traffic light (in Europe)

FSP model of a traffic light :

```
TRAFFICLIGHT = (red->orange->green->orange  
-> TRAFFICLIGHT).
```

LTS generated using *LTS*:



Trace:

red  $\rightarrow$  orange  $\rightarrow$  green  $\rightarrow$  orange  $\rightarrow$  red  $\rightarrow$  orange  $\rightarrow$  green ...

# FSP - choice

- If  $x$  and  $y$  are actions then

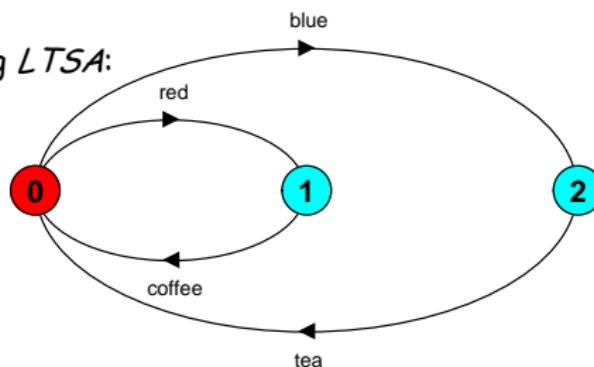
$$(x \rightarrow P \mid y \rightarrow Q)$$

describes a process which initially engages in either of the actions  $x$  or  $y$ . After the first action has occurred, the subsequent behavior is described by  $P$  if the first action was  $x$  and  $Q$  if the first action was  $y$ .

FSP model of a drinks machine :

```
DRINKS = (red->coffee->DRINKS
           |blue->tea->DRINKS
           ).
```

LTS generated using *LTS*A:



Possible traces?

# Non-deterministic choice

- Process  $(x \rightarrow P \mid x \rightarrow Q)$  describes a process which engages in  $x$  and then behaves as either  $P$  or  $Q$ .

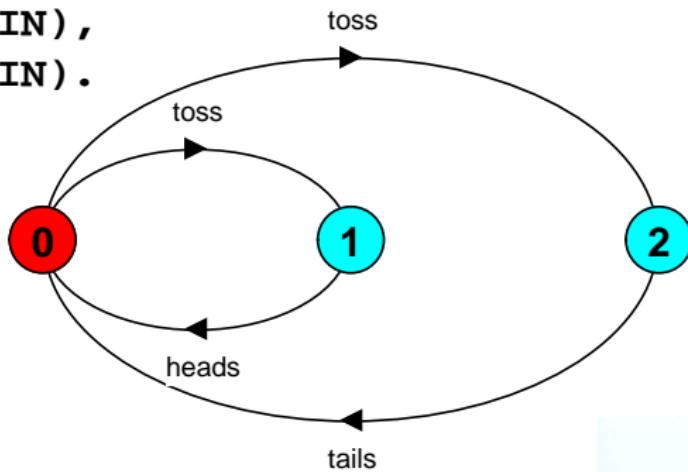
COIN = (toss->HEADS | toss->TAILS),

HEADS= (heads->COIN),

TAILS= (tails->COIN).

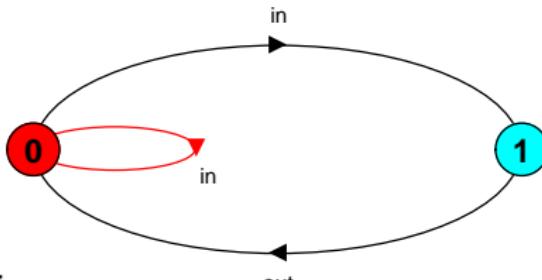
Tossing a  
coin.

Possible traces?



- How do we model an unreliable communication channel which accepts **in** actions and if a failure occurs produces no output, otherwise performs an **out** action?

Use non-determinism...



```
CHAN = (in->CHAN  
| in->out->CHAN  
).
```

- Single slot buffer that inputs a value in the range 0 to 3 and then outputs that value.

```
BUFF = (in[i:0..3]->out[i]-> BUFF).
```

equivalent to

```
BUFF = (in[0]->out[0]->BUFF  
| in[1]->out[1]->BUFF  
| in[2]->out[2]->BUFF  
| in[3]->out[3]->BUFF  
).
```

indexed actions  
generate labels of  
the form  
*action.index*

or using a **process parameter** with default value:

```
BUFF(N=3) = (in[i:0..N]->out[i]-> BUFF).
```

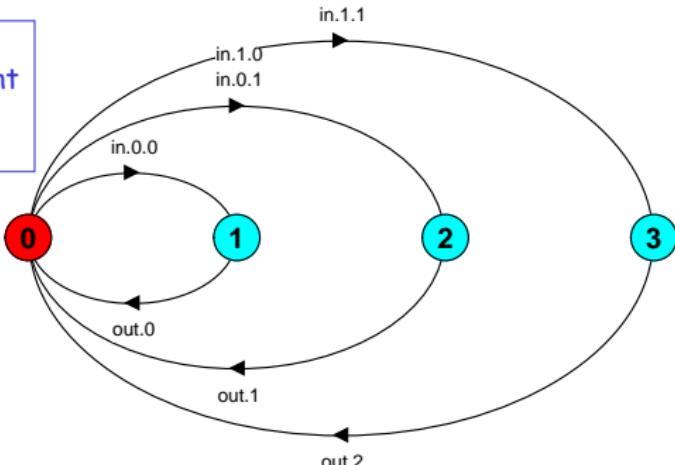
# More helpful syntax

Local indexed process definitions are equivalent to process definitions for each index value

index expressions to model calculation:

```
const N = 1
range T = 0..N
range R = 0..2*N
```

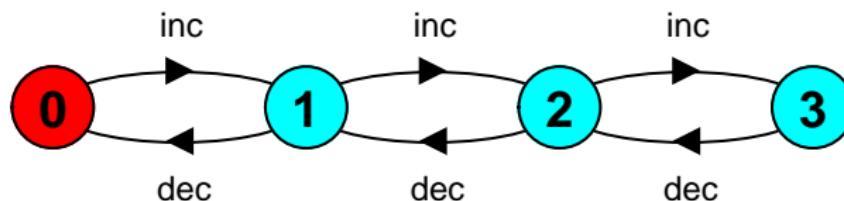
```
SUM      = (in[a:T][b:T] -> TOTAL[a+b]),
TOTAL[s:R] = (out[s] -> SUM).
```



- Notation:  $in[0][1] = in.0.1$ ,  $out[2] = out.2$ , etc.

- The choice ( $\text{when } B \ x \rightarrow P \mid y \rightarrow Q$ ) means that when the guard  $B$  is true then the actions  $x$  and  $y$  are both eligible to be chosen, otherwise if  $B$  is false then the action  $x$  cannot be chosen.

```
COUNT (N=3)      = COUNT[ 0 ] ,  
COUNT[i:0..N] = (when(i<N) inc->COUNT[i+1]  
                  | when(i>0) dec->COUNT[i-1]  
                  ).
```

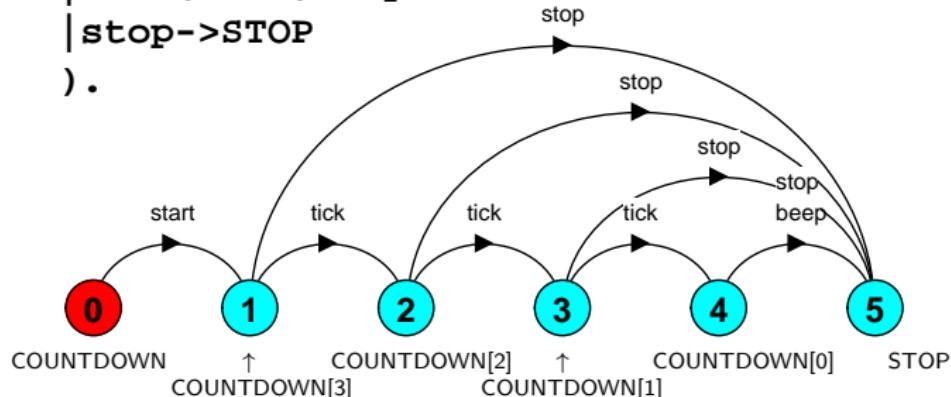


- It usually occurs in the form ( $\text{when } B \ x \rightarrow P \mid \text{when } \neg B \ y \rightarrow Q$ ), so the choice between  $x \rightarrow P$  and  $y \rightarrow Q$  is exclusive.

- A countdown timer which beeps after  $N$  ticks, or can be stopped.

```

COUNTDOWN (N=3)      = (start->COUNTDOWN[N]),
COUNTDOWN[i:0..N] =
  (when(i>0) tick->COUNTDOWN[i-1]
  | when(i==0) beep->STOP
  | stop->STOP
  ).
```



# Limitations of ( $when B \ x \rightarrow P \mid y \rightarrow Q$ )

- The domain of  $B$  must be **finite**. Otherwise we cannot create any LTS.

```
COUNT (N=3)      = COUNT[0],  
COUNT[i:0..N] = (when(i<N) inc->COUNT[i+1]  
                  | when(i>0) dec->COUNT[i-1]  
                  ).
```

- In this case the domains of  $(i < N)$  and  $(i > 0)$  consist of four elements  $i = 0, 1, 2, 3$ , so COUNT expands to:

$COUNT = COUNT0$

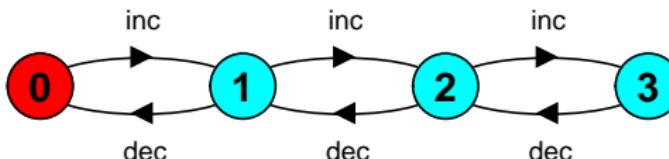
$COUNT0 = (inc \rightarrow COUNT1)$

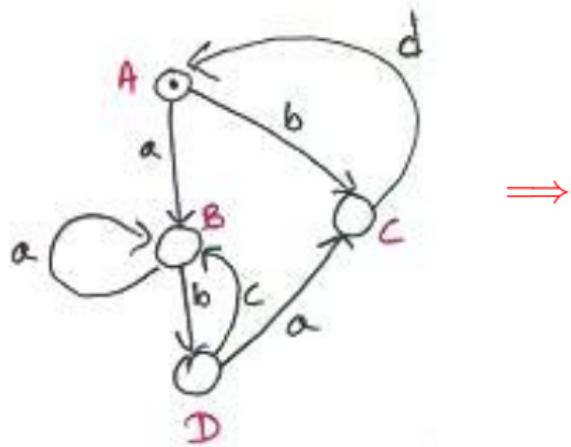
$COUNT1 = (inc \rightarrow COUNT2 \mid dec \rightarrow COUNT0)$

$COUNT2 = (inc \rightarrow COUNT3 \mid dec \rightarrow COUNT1)$

$COUNT3 = (dec \rightarrow COUNT2)$

- so, the states correspond to the values 0, 1, 2, 3, and LTS is:





$$\begin{aligned}A &= (a \rightarrow B \mid b \rightarrow C) \\B &= (a \rightarrow B \mid b \rightarrow D) \\C &= (d \rightarrow A) \\D &= (a \rightarrow C \mid c \rightarrow B)\end{aligned}$$

$$A = (a \rightarrow b \rightarrow B \mid b \rightarrow (a \rightarrow c \rightarrow A \mid b \rightarrow B))$$

$$B = (a \rightarrow c \rightarrow (a \rightarrow A \mid b \rightarrow b))$$

often some parentheses can be omitted for readability, i.e., we may write:

$$A = a \rightarrow b \rightarrow B \mid b \rightarrow (a \rightarrow c \rightarrow A \mid b \rightarrow B)$$

$$B = a \rightarrow c \rightarrow (a \rightarrow A \mid b \rightarrow B)$$

$$B_1 = b \rightarrow B$$



$$A_1 = c \rightarrow A$$

$$D_1 = a \rightarrow A_1 \mid b \rightarrow B \Rightarrow$$

```

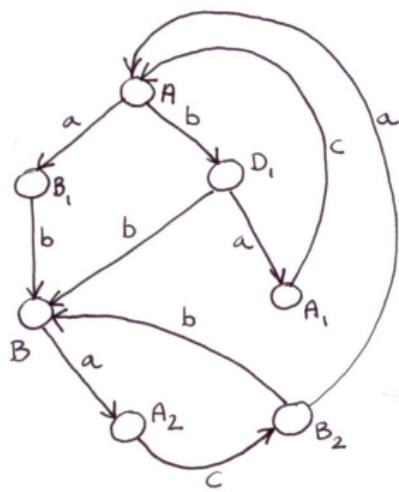
graph LR
    A1((A1)) -- a --> B1((B1))
    B1 -- b --> B((B))
    D1((D1)) -- b --> B
  
```

$$A = a \rightarrow B_1 \mid b \rightarrow D_1$$

$$B_2 = a \rightarrow A \mid b \rightarrow B$$

$$A_2 = c \rightarrow B_2$$

$$B = a \rightarrow A_2$$



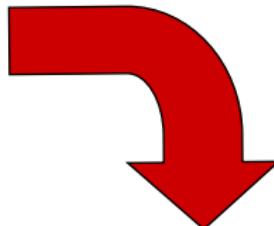
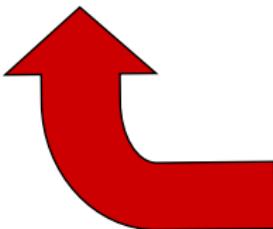
- The alphabet of a process is the set of actions in which it can engage.
- Process alphabets are **implicitly** defined by the actions in the process definition.
- Alphabet extension can be used to extend the implicit alphabet of a process:

$$\text{WRITER} = (\text{write}[1] \rightarrow \text{write}[3] \rightarrow \text{WRITER}) + \{\text{write}[0..3]\}$$

Alphabet of *WRITER* is the set

$$\{\text{write}[0..3]\} = \{\text{write}[0], \text{write}[1], \text{write}[2], \text{write}[3]\}.$$

Modeling **processes** as finite state machines using FSP/LTS.



Implementing **threads** in Java.

- Process  $\implies$  models as FSP or LTS
- Thread  $\implies$  implementation in Java

- **Concurrency:** Logically simultaneous processing. Does not imply multiple processing elements (PEs). Requires interleaved execution on a single PE.
- **Parallelism:** Physically simultaneous processing. Involves multiple PEs and/or independent device operations.

The textbook uses the terms parallel and concurrent interchangeably and generally do not distinguish between real and pseudo-concurrent execution.

- **These are the authors definitions!**
- **They are NOT universally accepted!**
- **WHAT ABOUT SIMULTANEITY AND SIMULTANEOUS EXECUTIONS?! They may make a substantial difference!**

- How should we model process execution speed?  
**Arbitrary speed (we abstract away time)**
- How do we model concurrency?  
**Arbitrary relative order of actions from different processes  
(interleaving but preservation of each process order)**

!!? MANY CONSIDER THIS APPROACH AS AN  
**OVERSIMPLIFICATION!**

- What is the result?  
**It provides a general model independent of scheduling  
(asynchronous model of execution)**

!!? MANY CONSIDER THE LAST STATEMENT AS AN  
**UNJUSTIFIED OVERSTATEMENT!**

# Parallel composition - action interleaving

- If  $P$  and  $Q$  are processes then  $(P||Q)$  represents the concurrent execution of  $P$  and  $Q$ . The operator  $||$  is the parallel composition operator.

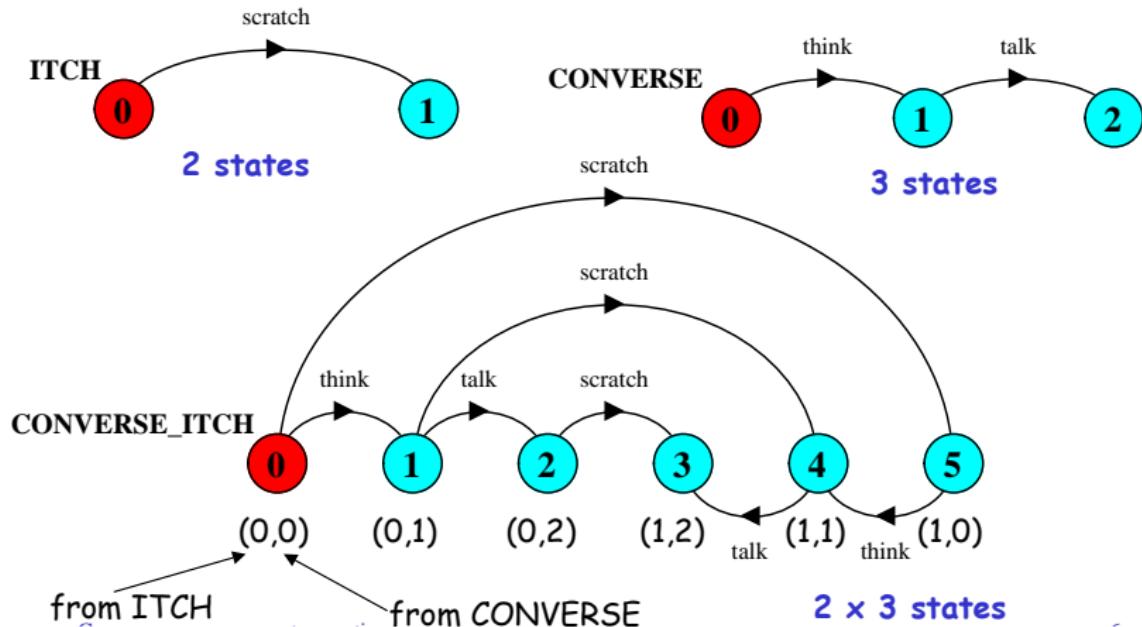
```
ITCH  = (scratch->STOP).  
CONVERSE = (think->talk->STOP).  
|| CONVERSE_ITCH = (ITCH || CONVERSE).
```

Disjoint alphabets

think → talk → scratch  
think → scratch → talk  
scratch → think → talk

Possible traces as  
a result of action  
interleaving.

# Parallel composition - action interleaving



- Transformation into LTS is NOT the best solution, transformation into *Petri nets* is better!

# Parallel composition: Algebraic Laws

- Commutativity:  $P \parallel Q = Q \parallel P$
- Associativity:  $P \parallel (Q \parallel R) = (P \parallel Q) \parallel R = P \parallel Q \parallel R$

## Problem: *What these equalities mean?*

- The set of traces that is generated is the same for the left and the right side, but is this sufficient?
- Semantics is not defined! In a decent scientific paper such "laws" would not survive!
- *Semantics should be defined before!*
- LTS are also the same for the left and right side of equations?  
Do they define semantics?

## Example (Clock Radio)

$CLOCK = tick \rightarrow CLOCK$

$RADIO = on \rightarrow off \rightarrow RADIO$

$\parallel CLOCK\_RADIO = CLOCK \parallel RADIO$

- If processes in a composition have actions in common, these actions are said to be *shared*. Shared actions are the way that process interaction is modeled. While unshared actions may be arbitrarily interleaved, a *shared action must be executed at the same time by all processes that participate in the shared action*.

## Example (Maker-user)

$MAKER = make \rightarrow \text{ready} \rightarrow MAKER$

$USER = \text{ready} \rightarrow use \rightarrow USER$

$\parallel MAKER\_USER = \text{Maker} \parallel USER$

## Traces:

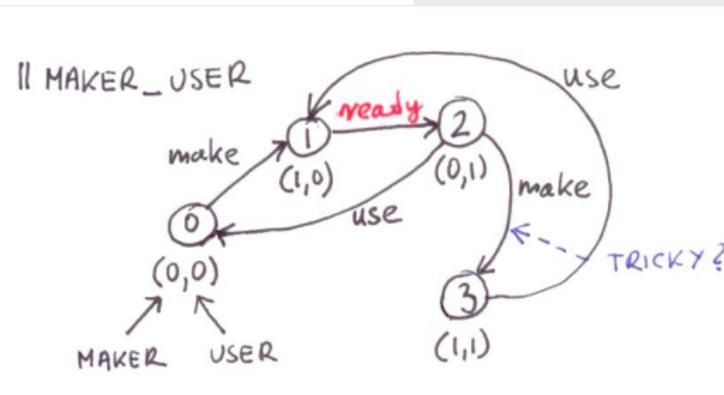
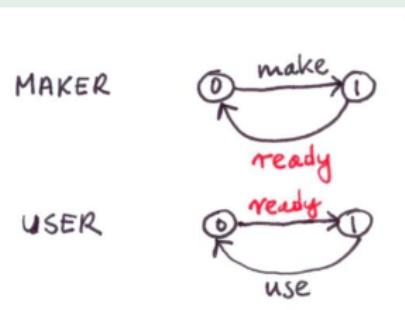
$make \rightarrow \text{ready} \rightarrow use \rightarrow make \rightarrow \text{ready} \rightarrow make \rightarrow use \rightarrow \dots$

## Example (Maker-user)

$MAKER = make \rightarrow ready \rightarrow MAKER$

$USER = ready \rightarrow use \rightarrow USER$

$\parallel MAKER\_USER = \text{Maker} \parallel \text{USER}$



- IT IS MUCH EASIER AND MORE INTUITIVE TO  
REPRESENT SYSTEMS LIKE MAKER-USER WITH *PETRI  
NETS!*