# Deadlock

## SE 3BB4

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# Deadlock

| | |
|---|---|
| Concepts: | system deadlock: no further progress<br>four necessary & sufficient conditions |
| Models: | deadlock - no eligible actions |
| Practice: | blocked threads |

Aim: deadlock avoidance - to design systems where deadlock cannot occur.

# Deadlock: four necessary and sufficient conditions

- **Serially reusable resources:**

  *the processes involved share resources which they use under mutual exclusion.*

- **Incremental acquisition:**

  *processes hold on to resources already allocated to them while waiting to acquire additional resources.*

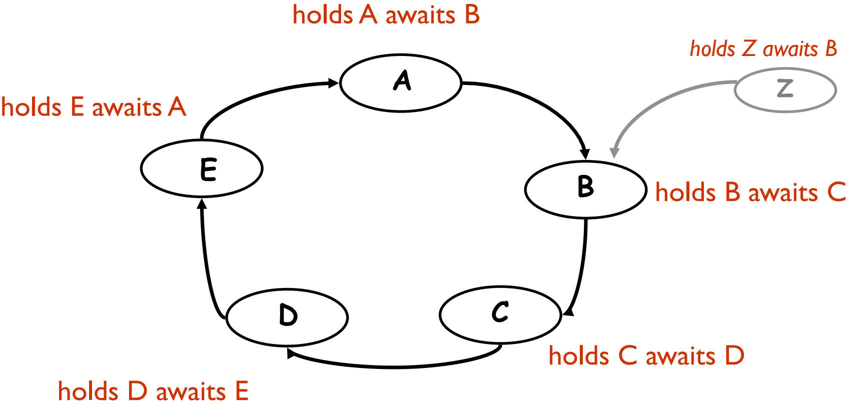
- **No pre-emption:**

  *once acquired by a process, resources cannot be pre-empted (forcibly withdrawn) but are only released voluntarily.*

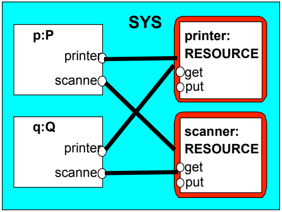
- **Wait-for cycle:**

  *a circular chain (or cycle) of processes exists such that each process holds a resource which its successor in the cycle is waiting to acquire.*

# Wait-for cycle



Deadlock may arise from the parallel composition of interacting processes.




```
RESOURCE = (get->put->RESOURCE).
P = (printer.get->scanner.get
      ->copy
      ->printer.put->scanner.put
      ->P).
Q = (scanner.get->printer.get
      ->copy
      ->scanner.put->printer.put
      ->Q).
||SYS = (p:P||q:Q
      ||{p,q}::printer:RESOURCE
      ||{p,q}::scanner:RESOURCE
      ).
```

**Deadlock Trace?**

**Avoidance?**

$$p : P = (p.printer.get_{\bullet} \rightarrow p.scanner.get \rightarrow p.copy \rightarrow p.printer.put \rightarrow p.scanner.put \rightarrow p : P)$$

$$q : Q = (q.scanner.get_{\bullet} \rightarrow q.printer.get \rightarrow q.copy \rightarrow q.scanner.put \rightarrow q.printer.put \rightarrow q : Q)$$

$$\underbrace{\{p, q\} :: printer : RESOURCE}_{pqpR} = (p.printer.get_{\bullet} \rightarrow p.printer.put \rightarrow pqpR \mid q.printer.get \rightarrow q.printer.put \rightarrow pqpR)$$

$$\underbrace{\{p, q\} :: scanner : RESOURCE}_{pqsR} = (p.scanner.get \rightarrow p.scanner.put \rightarrow pqsR \mid q.scanner.get_{\bullet} \rightarrow q.scanner.put \rightarrow pqsP)$$

**Deadlock sequence:** $p.printer.get \rightarrow q.scanner.get$

- $\bullet$ - denote states where processes deadlock

# A Possible Solutions

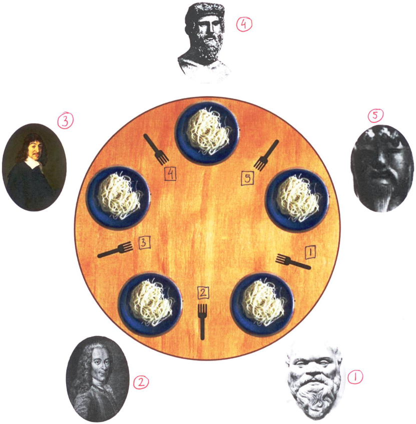
- Acquire resources in the same order, i.e. printers always before scanners.
- Timeout:

```
P          = (printer.get-> GETSCANNER),
GETSCANNER = (scanner.get->copy->printer.put
                                  ->scanner.put->P
             |timeout -> printer.put->P
             ).
Q          = (scanner.get-> GETPRINTER),
GETPRINTER = (printer.get->copy->printer.put
                                  ->scanner.put->Q
             |timeout -> scanner.put->Q
             ).
```

- No deadlock but the sequence:
  $printer.get \rightarrow timeout \rightarrow printer.put \rightarrow$
  can be repeated infinite number of times!
  NOBODY COPIES ANYTHING!

# Dining Philosophers

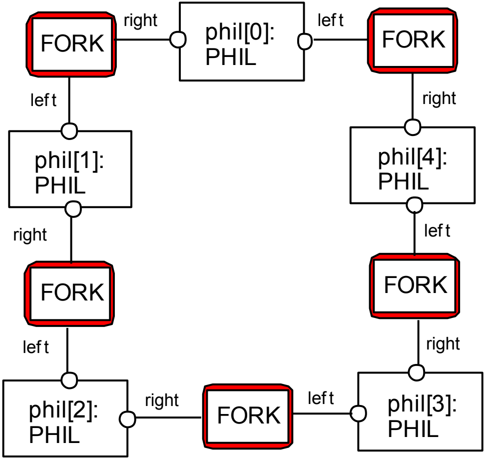
- Five philosophers sit around a circular table. Each philosopher spends his life alternately thinking and eating. To eat, a philosopher needs two forks, but unfortunately there are only five forks on the circulr table and each philosopher is only allowed to use the two forks nearest to him.



# Dining Philosophers - model structure diagram

Each FORK is a shared resource with actions **get** and **put**.

When hungry, each PHIL must first get his right and left forks before he can start eating.



# Hungry, Simple Minded Philosophers

- $i \oplus 1 = \textit{if } i < 5 \textit{ then } i+1 \textit{ else } 1$

$FORK = (get \rightarrow put \rightarrow FORK)$

$PHIL = (think \rightarrow right.get \rightarrow left.get \rightarrow eat \rightarrow right.put \rightarrow$
$left.put \rightarrow PHIL)$

$\| DINERS(N=5) = forall[i:1..N]$
$(phil[i]:PHIL \,\|\, \{phil[i].right, phil[i \oplus 1].left\} :: FORK)$

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- More intuitively (for $get_j^i, put_j^i$, $i$ - philosopher number, $j$ - fork number):

$FORK_1 = (get_1^1 \rightarrow put_1^1 \rightarrow FORK_1 \mid get_1^5 \rightarrow put_1^5 \rightarrow FORK_1)$
$FORK_2 = (get_2^2 \rightarrow put_2^2 \rightarrow FORK_2 \mid get_2^1 \rightarrow put_2^1 \rightarrow FORK_2)$
$FORK_3 = (get_3^3 \rightarrow put_3^3 \rightarrow FORK_3 \mid get_3^2 \rightarrow put_3^2 \rightarrow FORK_3)$
$FORK_4 = (get_4^4 \rightarrow put_4^4 \rightarrow FORK_4 \mid get_4^3 \rightarrow put_4^3 \rightarrow FORK_4)$
$FORK_5 = (get_5^5 \rightarrow put_5^5 \rightarrow FORK_5 \mid get_5^4 \rightarrow put_5^4 \rightarrow FORK_5)$
$PHIL_1 = (think_1 \rightarrow get_1^1 \rightarrow get_2^1 \rightarrow eat_1 \rightarrow put_1^1 \rightarrow put_2^1 \rightarrow PHIL_1)$
$PHIL_2 = (think_2 \rightarrow get_2^2 \rightarrow get_3^2 \rightarrow eat_2 \rightarrow put_2^2 \rightarrow put_3^2 \rightarrow PHIL_2)$
$PHIL_3 = (think_3 \rightarrow get_3^3 \rightarrow get_4^3 \rightarrow eat_3 \rightarrow put_3^3 \rightarrow put_4^3 \rightarrow PHIL_3)$
$PHIL_4 = (think_4 \rightarrow get_4^4 \rightarrow get_5^4 \rightarrow eat_4 \rightarrow put_4^4 \rightarrow put_5^4 \rightarrow PHIL_4)$
$PHIL_5 = (think_5 \rightarrow get_5^5 \rightarrow get_1^5 \rightarrow eat_5 \rightarrow put_5^5 \rightarrow put_1^5 \rightarrow PHIL_5)$
$\| DINERS = (FORK_1 \,\|\, \ldots \,\|\, FORK_5 \,\|\, PHIL_1 \,\|\, \ldots \,\|\, PHIL_5)$

- **Obvious deadlock! Everyone picks right fork.**

$Trace_1 =$

$phil.1.think \rightarrow phil.1.right.get \rightarrow$
$phil.2.think \rightarrow phil.2.right.get \rightarrow$
$phil.3.think \rightarrow phil.3.right.get \rightarrow$
$phil.4.think \rightarrow phil.4.right.get \rightarrow$
$phil.5.think \rightarrow phil.5.right.get$

$think_1 \rightarrow get_1^1 \rightarrow$
$think_2 \rightarrow get_2^1 \rightarrow$
$think_3 \rightarrow get_3^1 \rightarrow$
$think_4 \rightarrow get_4^1 \rightarrow$
$think_5 \rightarrow get_5^1$

# What if not 'Simple Minded'?

$$FORK = (get \rightarrow put \rightarrow FORK)$$

$$PHIL = THINK$$
$$THINK = (think \rightarrow$$
$$(right.get \rightarrow left.get \rightarrow EAT \mid left.get \rightarrow right.get \rightarrow EAT))$$

$$EAT = (eat \rightarrow$$
$$(right.put \rightarrow left.pt \rightarrow THINK \mid left.put \rightarrow right.put \rightarrow THINK$$

$$\| \, DINERS(N=5) = forall[i:1..N]$$
$$(phil[i] : PHIL \, \| \, \{phil[i].right, phil[i \oplus 1].left\} :: FORK)$$

- Unfortunately a freedom of choosing either right or left fork does not solve the problem. The same trace leads to a deadlock. However in "real" implementation, it will make it happen less often.

# Still 'Simple Minded' but not so 'Hungry'

$FORK = (get \to put \to FORK)$
$PHIL = THINK$
$THINK = (think \to right.get \to$
$\quad (left.get \to EAT \mid giveup \to right.put \to THINK))$
$EAT = (eat \to right.put \to left.put \to THINK)$
$\| DINERS(N = 5) = forall[i : 1..N]$
$\quad (phil[i] : PHIL \parallel \{phil[i].right, phil[i \oplus 1].left\} :: FORK)$

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- **There is no deadlock now!**
  $Trace_1 \to phil.i.giveup \to phil.i.right.put \to \ldots$
- However we might get:
  $Trace_1 \to Trace_2 \to$ and so on,
  where: $Trace_2 = phil.1.giveup \to phil.1.right.put \to$
  $phil.2.giveup \to phil.2.right.put \to$
  $phil.3.giveup \to phil.3.right.put \to$
  $phil.4.giveup \to phil.4.right.put \to$
  $phil.5.giveup \to phil.5.right.put \to$
- No philosopher will ever eat!
  **Starvation!**

# 'Hungry' and 'Asymmetrically Simple Minded', or 'Some Discipline Added'

- Philosophers 1, 3 and 5 always perform '$left.get \rightarrow right.get$', while 2 and 4 always perform '$right.get \rightarrow left.get$'.

$$FORK = (get \rightarrow put \rightarrow FORK)$$

$$\begin{aligned} PHIL = (&when(i = 1 \vee i = 3 \vee i = 5)\ think \rightarrow left.get \rightarrow \\ &right.get \rightarrow eat \rightarrow left.put \rightarrow right.put \rightarrow PHIL \\ &\mid when(i = 2 \vee i = 4)\ think \rightarrow right.get \rightarrow \\ &left.get \rightarrow eat \rightarrow right.put \rightarrow left.put \rightarrow PHIL) \end{aligned}$$

$$\begin{aligned} \| DINERS(N = 5) = forall[i : 1..N] \\ (phil[i] : PHIL \,\|\, \{phil[i].right, phil[i \oplus 1].left\} :: FORK) \end{aligned}$$

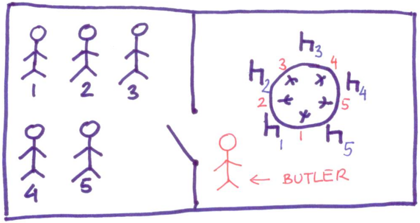
- **Works! Neither deadlock nor starvation.**

# Asymmetrically Simple Minded Philosophers

- Notation: for $get^i_j, put^i_j$, $i$ - philosopher number, $j$ - fork number

$$
\begin{aligned}
&FORK_1 = (get^1_1 \rightarrow put^1_1 \rightarrow FORK_1 \mid get^5_1 \rightarrow put^5_1 \rightarrow FORK_1)\\
&FORK_2 = (get^2_2 \rightarrow put^2_2 \rightarrow FORK_2 \mid get^1_2 \rightarrow put^1_2 \rightarrow FORK_2)\\
&FORK_3 = (get^3_3 \rightarrow put^3_3 \rightarrow FORK_3 \mid get^2_3 \rightarrow put^2_3 \rightarrow FORK_3)\\
&FORK_4 = (get^4_4 \rightarrow put^4_4 \rightarrow FORK_4 \mid get^3_4 \rightarrow put^3_4 \rightarrow FORK_4)\\
&FORK_5 = (get^5_5 \rightarrow put^5_5 \rightarrow FORK_5 \mid get^4_5 \rightarrow put^4_5 \rightarrow FORK_5)\\
&PHIL_1 = (think_1 \rightarrow get^1_2 \rightarrow get^1_1 \rightarrow eat_1 \rightarrow put^1_2 \rightarrow put^1_1 \rightarrow PHIL_1)\\
&PHIL_2 = (think_2 \rightarrow get^2_2 \rightarrow get^2_3 \rightarrow eat_2 \rightarrow put^2_2 \rightarrow put^2_3 \rightarrow PHIL_2)\\
&PHIL_3 = (think_3 \rightarrow get^3_4 \rightarrow get^3_3 \rightarrow eat_3 \rightarrow put^3_4 \rightarrow put^3_3 \rightarrow PHIL_3)\\
&PHIL_4 = (think_4 \rightarrow get^4_4 \rightarrow get^4_5 \rightarrow eat_4 \rightarrow put^4_4 \rightarrow put^4_5 \rightarrow PHIL_4)\\
&PHIL_5 = (think_5 \rightarrow get^5_1 \rightarrow get^5_5 \rightarrow eat_5 \rightarrow put^5_1 \rightarrow put^5_5 \rightarrow PHIL_5)\\
&\| \, DINERS = (FORK_1 \parallel \ldots \parallel FORK_5 \parallel PHIL_1 \parallel \ldots \parallel PHIL_5)
\end{aligned}
$$

- **No more than 4 philosophers are sitting at the table.**




$$FORK = (get \rightarrow put \rightarrow FORK)$$

$$PHIL = (think \rightarrow sitdown \rightarrow right.get \rightarrow left.get \rightarrow eat \rightarrow$$
$$right.put \rightarrow left.put \rightarrow getup \rightarrow PHIL)$$

$$BUTLER(K = 4) = COUNT[0]$$

$$COUNT[i : 1..4] = (when(i < K)\ sitdown \rightarrow COUNT[i+1] \mid$$
$$getup \rightarrow COUNT[i-1]$$

$$\| DINERS(N = 5) = (forall[i : 1..N]$$
$$(phil[i] : PHIL \,\|\, \{phil[i].right, phil[i \oplus 1].left\} :: FORK)$$
$$\| \quad \underbrace{\{phil[i : ..N]\}}_{\{phil[1], phil[2], phil[3], phil[4], phil[5]\}} \quad :: BUTLER(K = 4))$$

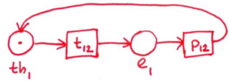
# 'Butler' Solution

- 'Butler' solution works. No deadlock and no starvation.
- *FORK*'s are *passive* processes (monitors), hence they always can be presented as:
  $FORK = (get \rightarrow put \rightarrow FORK)$
- *PHILOSOPHER*'s are *active* processes.

# A Solution with Simultaneity

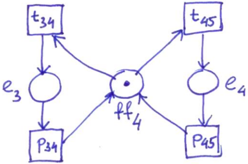
- No philosopher is allowed to grab one fork only, he must take both left and right at the same time if they are available.
- Modeling simultaneity is **not** *natural* in FSP approach, it is possible but looks artificial.
- Modeling simultaneity is *natural* when Petri nets are used.

- Philosopher No. 1:



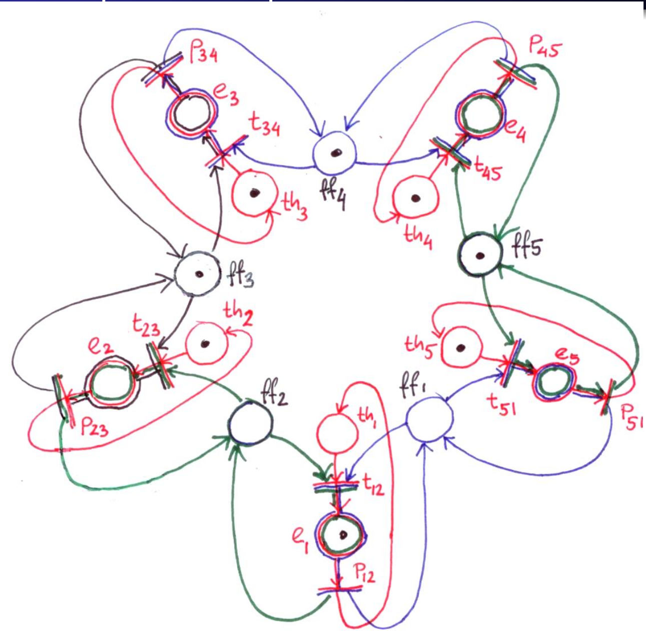

$tt_1$ - Philosopher No.1 thinks
$e_1$ - Philosopher No.1 eats
$t_{12}$ - Philosopher No.1 takes up forks 1 and 2
$p_{12}$ - Philosopher No.1 puts down forks 1 and 2

- Free Fork No. 4:




$ff_4$ - Fork No. 4 is on the table
$e_3$ - Philosopher No. 3 eats using Fork No. 4
$t_{34}$ - Fork No. 4 is taken by Philosopher No.3
$p_{34}$ - Fork No. 4 is put down Philosopher No.3
$e_4$ - Philosopher No. 4 eats using Fork No. 4
$t_{45}$ - Fork No. 4 is taken by Philosopher No. 4
$p_{45}$ - Fork No. 4 is put down Philosopher No. 4

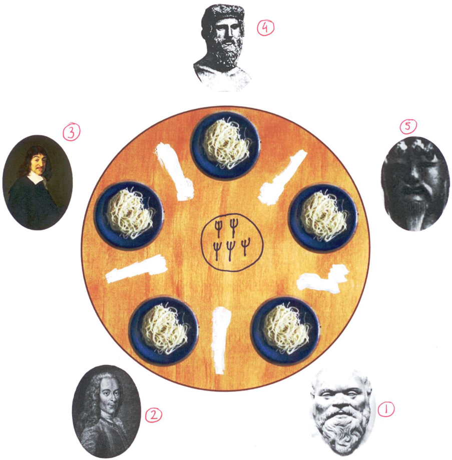
# Dining Philosophers Composed



# A Solution with Simultaneity for FSP

$$FORK = (take.right \to put.right \to FORK \mid$$
$$take.left \to put.left \to FORK)$$
$$PHIL = (think \to takeboth \to eat \to putboth \to PHIL)$$

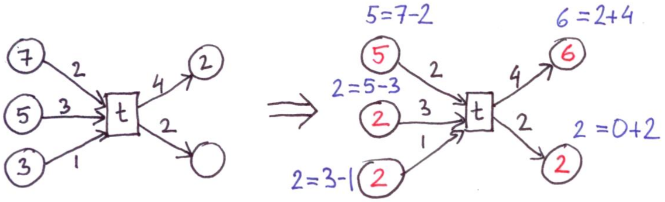
$$\| DINERS(N = 5) = forall[i : 1..N]$$
$$(phil[i] : PHIL \| \{phil[i].right, phil[i \oplus 1].left\} :: FORK))$$
$$/\{takeboth.1/take.right.1, takeboth.1/take.left.2,$$
$$takeboth.2/take.right.2, takeboth.2/take.left.3,$$
$$takeboth.3/take.right.3, takeboth.3/take.left.4,$$
$$takeboth.4/take.right.4, takeboth.4/take.left.5,$$
$$takeboth.5/take.right.5, takeboth.5/take.left.1,$$
$$putboth.1/put.right.1, putboth.1/put.left.2,$$
$$putboth.2/put.right.2, putboth.1/put.left.3,$$
$$putboth.3/put.right.3, putboth.3/put.left.4,$$
$$putboth.4/put.right.4, putboth.4/put.left.5,$$
$$putboth.5/put.right.5, putboth.5/put.left.1\}$$



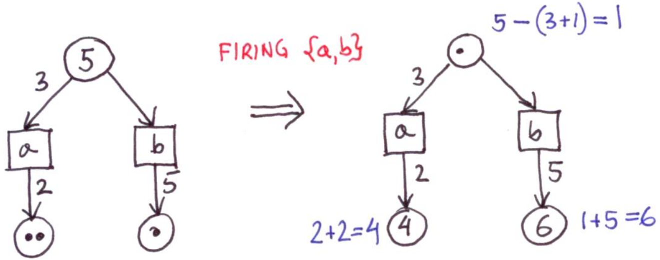

- All forks in one bowl. Forks are not distinguishable and philosophers pick them randomly.

# Place/Transitions Nets (P/T-Nets)

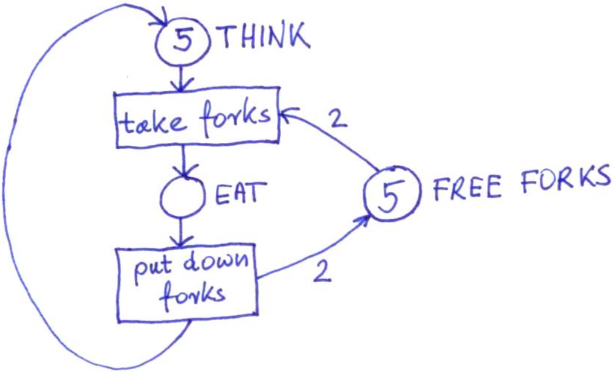
- Firing rules:



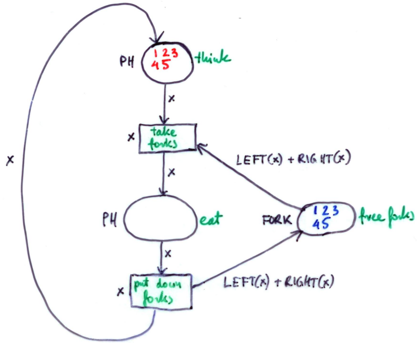

- Different kind of simultaneity:



# P/T-Nets Solution to 2nd Dining Philosophers Problem




- It does not work for the original Dining Philosophers Problem.
- Both philosophers and forks are represented by tokens.
- State machines represent *generic behaviours*.
- Impossible to model directly with FSP.




$colour\ PH = with\ ph1 \mid ph2 \mid ph3 \mid ph4 \mid ph5$

$colour\ Fork = with\ f1 \mid f2 \mid f3 \mid f4 \mid f5$

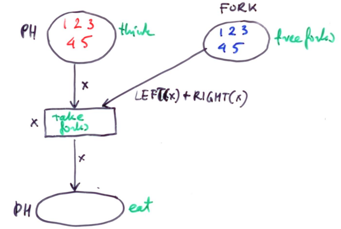
$LEFT : PH \rightarrow FORK, \quad RIGHT : PH \rightarrow FORK$

$var\ x :\ PH$

$fun\ LEFT\ x = case\ of\ ph1 \Rightarrow f2 \mid ph2 \Rightarrow f3 \mid ph3 \Rightarrow f4 \mid$
$ph4 \Rightarrow f5 \mid ph5 \Rightarrow f1$

$fun\ RIGHT\ x = case\ of\ ph1 \Rightarrow f1 \mid ph2 \Rightarrow f2 \mid ph3 \Rightarrow f3 \mid$
$ph4 \Rightarrow f4 \mid ph5 \Rightarrow f5$

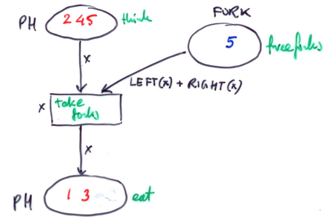
# Firing




$$\Downarrow$$

Firing occurrence: $(take\ forks, \underbrace{x = ph1}_{binding}) + (take\ forks, \underbrace{x = ph3}_{binding})$

$$\Downarrow$$



# Multisets (or Bags)

- A multiset $m$, over a non-empty and finite set $S$ is a function $m : S \to \mathbb{N} = \{0, 1, 2, \ldots\}$
- $m(s)$ is the number of appearances of $s$ in $m$.
- notation: $M$ is usually represented by:

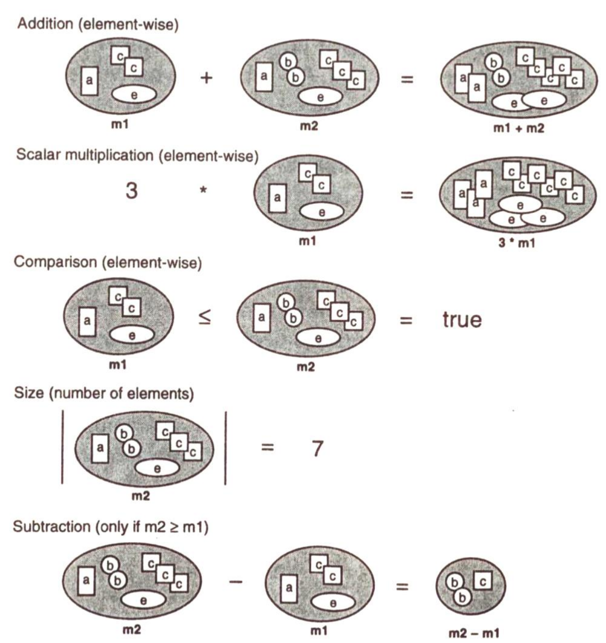
$$\sum_{s \in S} m(s)s$$

$S = \{a, b, c, d, e\}$,
$m(a) = 3, m(b) = 1, m(c) = 0, m(d) = 183, m(e) = 4$

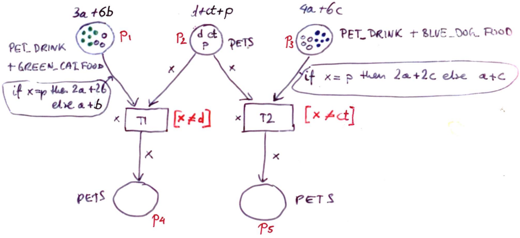
$$m = 3a + b + 183d + 4e$$

- $s \in m \iff m(s) \neq 0$
- $m(s)$ is a *coefficient*
- the *empty multiset* $m = \emptyset \iff m(s)$ for each $s \in S$.

# Some Operations on Multisets

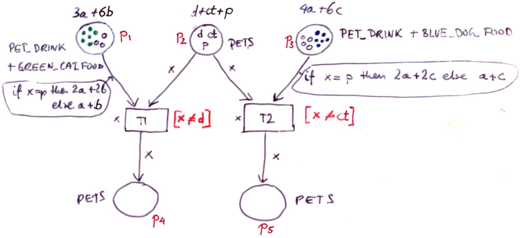


*colour PET_DRINK = with a;*
*colour GREEN_CAT_FOOD = with b;*
*colour BLUE_DOG_FOOD = with c;*
*colour PETS = with dog | cat | pig;* (pig eats both cat food and dog food)
*var x : PETS;* (in the drawing: dog=d, cat =ct, pig = p)




• In place $p_1$ we have 6 green cat food servings and 3 drinks (no dog food as it not allowed here, it is allowed in place $p_3$)
• Firing transition $T1$ corresponds to allow cat or pig or both to eat. The cat eats 1 serving of cat food and 1 drink while the pig eats 2 servings of food and 2 drinks.
• If both cat and pig eat and drink, 3 drinks and 3 servings of cat food disappear from place $p_1$, and $p, ct$ disappear from place $p_2$.
• Similarly for places $p_3, p_5$ and transition $T2$.

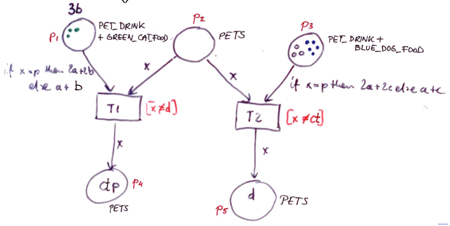
*colour* $PET\_DRINK$ = *with* $a$;
*colour* $GREEN\_CAT\_FOOD$ = *with* $b$;
*colour* $BLUE\_DOG\_FOOD$ = *with* $c$;
*colour* $PETS$ = *with* $dog \mid cat \mid pig$; (pig eats both cat food and dog food)
*var* $x$ : $PETS$; (in the drawing: dog=d, cat =ct, pig = p)



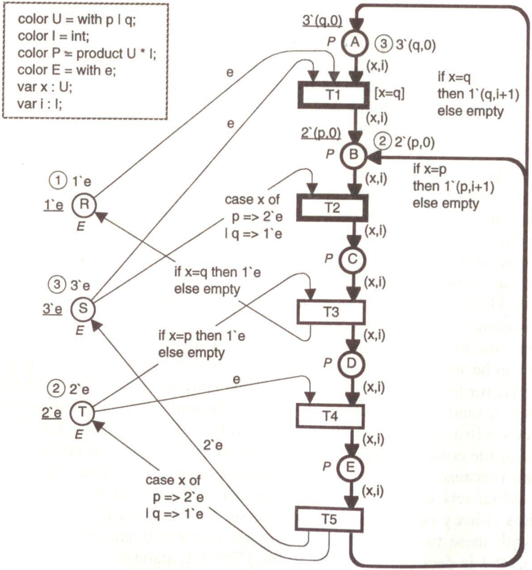

⇓

Firing occurrence:$(T1, x = c) + (T1, x = p) + (T2, x = d)$
Interpretation:All three pets eat.

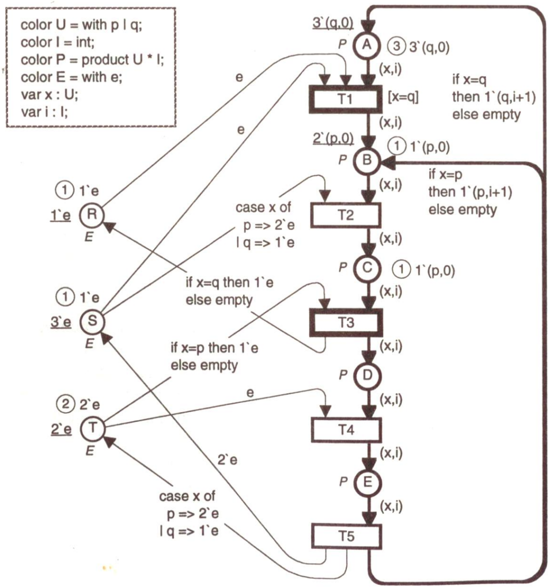
⇓



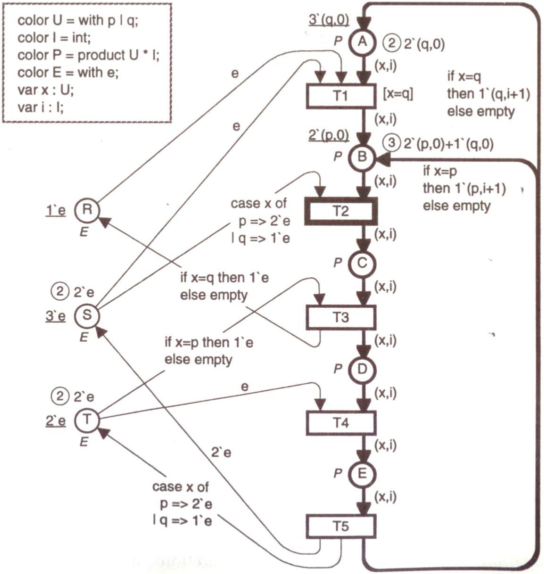
# Some resource allocation syatem in its initial marking $M_0$



# $M_1$ reachable from $M_0$ by $(T2, \langle x = p, i = 0 \rangle)$



# $M_2$ reachable from $M_0$ by $(T2, \langle x = q, i = 0 \rangle)$



A *Coloured Petri Net* is a tuple: $N = (P, T, A, \Sigma, C, N, E, G, I)$ where:

- $P$ is a set of places and $T$ is a set of transitions.
- $T$ is a set of transitions.
- $A$ is a set of arcs
- In CPNs sets of places, transitions and arcs are pairwise disjoint $P \cap T = P \cap A = T \cap A = \emptyset$
- $\Sigma$ is a set of color sets defined within CPN model. This set contains all possible colors, operations and functions used within CPN.
- $C$ is a colour function. It maps places in $P$ into colors in $\Sigma$.
- $N$ is a node function. It maps $A$ into $(P \times T) \cup (T \times P)$.
- $E$ is an arc expression function. It maps each arc $a \in A$ into the expression $e$. The input and output types of the arc expressions must correspond to type of nodes the arc connected to.
- $G$ is a guard function. It maps each transition $t \in T$ into guard expression $g$. The output of the guard expression should evaluate to Boolean value true or false.
- $I$ is an initialization function. It maps each place $p$ into an initialization expression $i$. The initialization expression must evaluate to multiset of tokens with a color corresponding to the

# Some Concepts Needed

- The *elements of a type*, T. The set of all elements in T is denoted by the type name T itself.
- The *type of a variable*, v – denoted by Type(v).
- The *type of an expression*, expr – denoted by Type(expr).
- The *set of variables in an expression*, expr – denoted by Var(expr).
- A *binding of a set of variables*, V – associating with each variable $v \in V$ an element $b(v) \in \text{Type}(v)$.
- The *value obtained by evaluating an expression*, expr, *in a binding*, b – denoted by expr<b>. Var(expr) is required to be a subset of the variables of b, and the evaluation is performed by substituting for each variable $v \in \text{Var(expr)}$ the value $b(v) \in \text{Type}(v)$ determined by the binding.

# 'Official' Formal Definition of Petri Nets

**Definition 2.5**: A **non-hierarchical CP-net** is a tuple CPN = (Σ, P, T, A, N, C, G, E, I) satisfying the requirements below:

(i) Σ is a finite set of non-empty types, called **colour sets**.
(ii) P is a finite set of **places**.
(iii) T is a finite set of **transitions**.
(iv) A is a finite set of **arcs** such that:
- $P \cap T = P \cap A = T \cap A = \emptyset$.

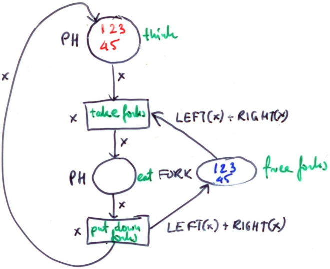
(v) N is a **node** function. It is defined from A into $P \times T \cup T \times P$.
(vi) C is a **colour** function. It is defined from P into Σ.
(vii) G is a **guard** function. It is defined from T into expressions such that:
- $\forall t \in T: [Type(G(t)) = \mathbb{B} \wedge Type(Var(G(t))) \subseteq \Sigma]$.

(viii) E is an **arc expression** function. It is defined from A into expressions such that:
- $\forall a \in A: [Type(E(a)) = C(p(a))_{MS} \wedge Type(Var(E(a))) \subseteq \Sigma]$

where p(a) is the place of N(a).

(ix) I is an **initialization** function. It is defined from P into closed expressions such that:
- $\forall p \in P: [Type(I(p)) = C(p)_{MS}]$.

# Behaviours




Sequence:

$(take\ forks, x = ph1)(take\ forks, x = ph3)(putdown\ forks, x = ph3)$

Step-sequence:

$\{(take\ forks, x = ph1)(take\ forks, x = ph3)\}\{(putdown\ forks, x = ph3)\}$

Partial order: