

Assignment 1 Solutions

1. The machine is given as Figure 2.3 in the textbook.
2. **(a)** The machine does not visit this square until the next crossing to the left, so it cannot change it.
(b) The reason is that the sum of the length of the different crossing sequences of each square in 0^n has to be bounded above by $T_M(x)$, and therefore not all squares can have crossing sequences that are very long. Let S be this short crossing sequence.
(c) The reason that we know that we have x is that suppose that the same holds for some $y \neq x$; then M would accept $y0^n x^R$ which is *not* a palindrome, and we assume that M works correctly.
(d) So we know if the machine accepted or rejected.
(e) Otherwise, we would have a bijection from the set of strings of length n to the set of strings of length $n - 1$, and that is not possible.
(f) Follows from the definition of Ω .
3. Theorem 2.1 in the textbook.
4. Here is an outline of the proof that a two tape linear time nondeterministic TM, M_2 , can simulate a k -tape such machine, M_k . Let's assume that the input to M_2 is written on tape 1. M_2 starts by guessing the entire computation of M_k , and writing the guess on tape 2. This computation does not contain the tape configurations of M_k , but rather at each step of M_k it writes down the (supposed) state and symbols scanned by each of the k heads. Note that this information (if correct) is enough to determine the next move of M_k .

Now M_2 checks that the computation it has written is correct, as follows. First it checks that the state sequence is consistent, by making one pass on tape 2, and checking that at each step the next state is what it should be, given the information from the previous step. Next, for each tape i of M_k , it makes a pass on tape 2 and checks that the scanned symbol information it has written for tape i is consistent. It does this by using tape 1 to simulate tape i , and using the information it has written on tape 2 concerning what the other $k - 1$ tapes are scanning.

If all these consistency checks are passed, then the information on tape 2 is correct (show that the t -th step is correct by induction on t). Hence M_2 can figure out what M_k did.