Assignment 4 **CAS705 Fall 2005** **Due on November 7th**

The work you submit must be your own. *You may discuss problems with each other; however, you should prepare written solutions alone. In particular, you should not leave with any written notes from such discussions. The style and clarity of your answers will be an important factor in the grade.*

Each question is worth 25%.

1. Consider the function pad : $\Sigma^* \times \mathbb{N} \longrightarrow (\Sigma \cup \{\#\})^*$ that is defined as follows. Let $\mathrm{pad}(s, l) = s\#^j$ where $j = \max(0, l - m)$, for $m = |s|$. For any language L and function $f : \mathbb{N} \longrightarrow \mathbb{N}$ define $\mathrm{pad}(L, f(n))$ as follows:

$$\{\mathrm{pad}(s, f(n)) |\text{ where } s \in L \text{ and } n = |s|\ \}$$

   (a) Prove that if $L \in \mathrm{TIME}(n^6)$, then $\mathrm{pad}(L, n^2) \in \mathrm{TIME}(n^3)$.

   (b) Prove that if **NEXPTIME** $\neq$ **EXPTIME**, then **P** $\neq$ **NP**.

2. Prove the **Time Hierarchy Theorem**, that is, given a time constructible function $t : \mathbb{N} \longrightarrow \mathbb{N}$, there exists a language A that is decidable in time $O(t(n))$ but not in time $o(t(n)/\log(t(n)))$. Conclude that $\mathbf{P} \subsetneq \mathbf{EXPTIME}$.

   A function $t : \mathbb{N} \longrightarrow \mathbb{N}$, where $t(n) \geq n\log(n)$ is called *time constructible* if the function that maps 1^n to the binary representation of $t(n)$ is computable in time $O(t(n))$.

3. Prove that an oracle C exists for which $\mathbf{NP}^C \neq \textbf{co-NP}^C$.

4. Define $\Sigma_0^{\mathrm{lin}} = \mathbf{LINTIME}$ and $\Sigma_{i+1}^{\mathrm{lin}} = \mathbf{NLINTIME}^{\Sigma_i^{\mathrm{lin}}}$. Here $\mathbf{NLINTIME}^{\Sigma_i^{\mathrm{lin}}}$ denotes the class of languages decidable in nondeterministic linear time with a Σ_i^{lin} oracle. Define the linear time hierarchy as $\mathbf{LTH} = \bigcup_i \Sigma_i^{\mathrm{lin}}$.

   Let $\mathrm{NTIMESPACE}(f(n), g(n))$ be the class of languages decided simultaneously in time $O(f(n))$ and space $O(g(n))$ on a nondeterministic (multi-tape) Turing machine.

   Let $0 < \varepsilon < 1$ be a rational number, and let a be a positive integer. Show that:

$$\mathrm{NTIMESPACE}(n^a, n^\varepsilon) \subseteq \mathbf{LTH}.$$