

The work you submit must be your own. You may discuss problems with each other; however, you should prepare written solutions alone. In particular, you should not leave with any written notes from such discussions. The style and clarity of your answers will be an important factor in the grade.

The following questions are problems from chapter 11 in Papadimitriou.

1. Show that **RP**, **BPP**, **PP** are closed under (log-space) reductions. [11.5.13]
2. Show that **BPP** and **RP** are closed under union and intersection. Also show that **PP** is closed under complement and symmetric difference. [11.5.14 and 15]
3. Show that MAJSAT (consisting of ϕ 's for which at least half of the truth assignments to $\text{Var}(\phi)$ are satisfying) is **PP**-complete. [11.5.16]
4. Show that if $\mathbf{NP} \subseteq \mathbf{BPP}$ then $\mathbf{RP} = \mathbf{NP}$. [11.5.18]
5. Suppose that a Boolean expression has fewer than n^k clauses, each with at least $k \log(n)$ distinct variables. Show that it must have a satisfying truth assignment. Then give a polytime algorithm for finding such a truth assignment. [11.5.23]
6. Question 11.5.24, (a),(b),(c), and (d).