Name _____ Student No. _____

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks **60**
Three questions, each question is worth **20**

1. Consider the Boolean array $R(i, j)$ in the dynamic programming solution to the simple knapsack problem.

   (a) Present the recurrence for computing $R(i, j)$.

   (b) Suppose that all the entries have been computed already; show how to use the table for R to give the *actual* set of weights.

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2. Consider the following greedy algorithm for solving the simple knapsack problem: order the weights from the heaviest to the lightest. Keep adding weights in that order for as long as they fit.

   (a) Give an example of a set of weights for which this greedy algorithm computes the optimal sum, and another set of weights for which it does not.

   (b) Suppose that $\overline{M}$ is the solution of the greedy algorithm. Show that either $\overline{M}$ is the optimal solution M, or $\overline{M} > \frac{1}{2}C$.

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3. This question concerns the randomized algorithm for perfect matching.

   (a) Present the Monte Carlo algorithm for perfect matching.

   (b) Consider the following graph:

$$G = (\{1, 2, 3\} \cup \{1', 2', 3'\}, \{(1, 2'), (2, 2'), (2, 3'), (3, 1')\}).$$

      i. Give A_G.

      ii. Give $\det(A_G)$.

      iii. Give an example of a random selection of integers in the algorithm, so that the algorithm outputs a false negative.

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End of Test 2