1. We show that $\forall i \in [n-1]$ it is the case that $r_{i+1} \sum_{j=1}^{i} r_j$ by induction on i. The basis case is $i-1$, so

$$r_2 \geq 2r_1 > r_1 = \sum_{j=1}^{i} r_j,$$

where $r_2 \geq 2r_1$ by the property of being super-increasing. For the induction step we have

$$r_{i+1} \geq 2r_i = r_i + r_i > r_i + \sum_{j=1}^{i-1} r_j = \sum_{j=1}^{i} r_j,$$

where we used the property of being super-increasing and the induction hypothesis.

2. Here is the algorithm:

$X \longleftarrow S$
$Y \longleftarrow \emptyset$
for $i = n \ldots 1$
    if $(r_i \leq X)$
        $X \longleftarrow X - r_i$
        $Y \longleftarrow Y \cup \{i\}$

and let the pre-condition state that $\{r_i\}_{i=1}^{n}$ is super-increasing and that there exists an $\mathcal{S} \subseteq \{r_i\}_{i=1}^{n}$ such that $\sum_{i \in \mathcal{S}} r_i = S$. Let the post-condition state that $\sum_{i \in Y} r_i = S$.

Define the following loop invariant: "Y is promising" in the sense that it can be extended, with indices of weights not considered yet, into a solution. That is, after considering i, there exists a subset E of $\{i-1, \ldots, 1\}$ such that $\sum_{j \in X \cup E} r_j = S$.

The basis case is trivial since initially $X = \emptyset$, and since the pre-condition guarantees the existence of a solution, X can be extended into that solution.

For the induction step, consider two cases. If $r_i > X$ then i is not added, but Y can be extended with $E' \subseteq \{i-1, i-2, \ldots, 1\}$. The reason is that by induction hypothesis X was extended into a solution by some $E \subseteq \{i, i-1, \ldots, 1\}$ and i was not part of the extension as r_i was too big to fit with what was already in Y, i.e., $E' = E$.

If $r_i \leq X$ then $i \in E$ since by Q1 the remaining weights would not be able to close the gap between S and $\sum_{j \in Y} r_j$.