

Name _____ Student No. _____

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: **60**

There are two empty pages after each question.

The test consists of 6 pages.

- [30] 1. Show the correctness of Euclid's algorithm, i.e., partial correctness and termination.

Algorithm 1 Euclid

Pre-condition: $a > 0 \wedge b > 0$

1: $m \leftarrow a ; n \leftarrow b ; r \leftarrow \text{rem}(m, n)$

2: **while** ($r > 0$) **do**

3: $m \leftarrow n ; n \leftarrow r ; r \leftarrow \text{rem}(m, n)$

4: **end while**

5: **return** n

Post-condition: $n = \text{gcd}(a, b)$

This is algorithm 1.2 in the textbook, section 1.3.2.

- [30] 2. Consider the Gale-Shapley algorithm; show that it produces a stable matching.
(**Hint:** define a *blocking pair*, and show that M_n has no blocking pairs.)
Finally, argue that the algorithm terminates.

Algorithm 2 Gale-Shapley

Stage 1: At stage 1, b_1 chooses the first girl g in his preference list and we set $M_1 = \{(b_1, g)\}$.

Stage $s + 1$:

$M \leftarrow M_s$

$b^* \leftarrow b_{s+1}$

Then b^* proposes to the girls in order of his preference until one accepts; girl g will accept the proposal as long as she is either not engaged or prefers b^* to her current partner $p_M(g)$.

Then we add (b^*, g) to M and proceed according to one of the following two cases:

(i) If g was not engaged, then we terminate the procedure and set

$M_{s+1} \leftarrow M \cup \{(b^*, g)\}$.

(ii) If g was engaged to b , then we set

$M \leftarrow (M - \{(b, g)\}) \cup \{(b^*, g)\}$

$b^* \leftarrow b$

and repeat.

This is algorithm 1.12 in the textbook, section 1.4.

End of Test