

1. Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. The alphabet is always $\Sigma = \{0, 1\}$.
 - (a) $\{w \mid w \text{ ends with } 00\}$ with three states.
 - (b) $1^*(001^+)^*$, where $L(R^+) = L(R^*) - \{\varepsilon\}$, with three states.
2. If L is a language its complement is denoted L^c and $L^c = \Sigma^* - L$, that is, L^c consists of all the strings in Σ^* that are not in L . Show that if L is regular then so is L^c .
3. Given a language L , $L^+ = \{w_1w_2 \cdots w_k \mid w_i \in L \text{ and } k \geq 1\}$. Prove that $L = L^+$ if and only if $L \circ L \subseteq L$.
4. Show that the following two languages are *not* regular.
 - (a) $L_1 = \{1^{n^2} \mid n \geq 0\} \subseteq \{1\}^*$. That is, L_1 is the set of unary strings (strings consisting only of 1s) whose length is a perfect square.
 - (b) $L_2 = \{0^i1^j \mid i > j\} \subseteq \{0, 1\}^*$. That is, L_2 is the set of binary strings consisting of a block of 0s followed by a block of 1s, where there are more 0s than 1s.