

1. The first two questions are about Turing machines and decidability. Let L be a language consisting of a single string w , i.e., $L = \{w\}$. The string w is defined as follows:

$$w = \begin{cases} 111 & \text{if there is life on Mars} \\ 000 & \text{if there is no life on Mars} \end{cases}.$$

Is L a *decidable* language? Justify your answer (assuming that the question “is there life on Mars?” has a definitive ‘yes’ or ‘no’ answer).

2. Show that decidable languages are closed under the following operations:
 - (a) union
 - (b) concatenation
 - (c) star
 - (d) complementation
 - (e) intersection
3. This is a λ -calculus question. Apply β -reductions to each of the following λ -terms until no more β -reductions can be applied:
 - (a) $(\lambda x.xx)(\lambda y.yx)z$
 - (b) $(\lambda x.(\lambda y.(xy))y)z$
 - (c) $((\lambda x.xx)(\lambda y.y))(\lambda y.y)$
 - (d) $((\lambda x.\lambda y(xy))(\lambda y.y))w$
4. This is a question about relations. Show that if R_1 and R_2 are relations such that $R_1 \subseteq R_2$, then $\text{index}(R_1) \geq \text{index}(R_2)$.