Assignment 3 **Due: Nov 20** **cs2mj3 Fall 2009**

For each natural number x, define $F(x)$ to be the number of distinct primes dividing x. For example, $F(100) = 2$ because there are precisely two distinct primes dividing 100, namely 2 and 5. Show, using the outline of questions below, that F is primitive recursive.

1. Show first that bounded sum and bounded product are primitive recursive, that is, show that g, h:
$$g(\vec{x}, y) = \sum_{z<y} f(\vec{x}, z) \quad \text{and} \quad h(\vec{x}, y) = \prod_{z<y} f(\vec{x}, z)$$
are primitive recursive if f is primitive recursive (here $\vec{x} = x_1, x_2, \ldots, x_n$).

2. Show that bounded quantification is primitive recursive, that is, show that S, T:
$$S(\vec{x}, y) = \exists z(z < y \wedge R(\vec{x}, z)) \quad \text{and} \quad T(\vec{x}, y) = \forall z(z < y \supset R(\vec{x}, z))$$
are primitive recursive if R is primitive recursive.

3. Show that the divisibility predicate is primitive recursive, that is $x|y$ (y is divisible by x) is primitive recursive.

4. Define the predicate Prime(x) to be 0 if x is prime and 1 if it is not. Show that Prime(x) is primitive recursive.

5. Finally, show that the function $G(x, y)$, defined to be the number of distinct primes dividing x which are less than or equal to y, is primitive recursive.

6. Now you are ready to show that F is primitive recursive.