

CMS 2MJ3: Theory of Computation

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This is problem 3.11 from Sipser's book "Introduction of Theory of Computation", page 161.

Problem: A Turing machine with doubly infinite tape is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it move leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.

Answer: A TM with doubly infinite tape can simulate an ordinary TM. It marks the left-hand end of the input to detect and prevent the head from moving off of that end. To simulate the doubly infinite tape TM by an ordinary TM, we show how to simulate it with a 2-tape TM, which was already shown to be equivalent in power to an ordinary TM. The first tape of the 2-tape TM is written with the input string and the second tape is blank. We cut the tape of the doubly infinite tape TM into two parts, at the starting cell of the input string. The portion with the input string and all the blank spaces to its right appears on the first tape of the 2-tape TM. The portion to the left of the input string appears on the second tape, in reverse order.