

**Ulam Seminar  
Computational Complexity**

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List of topics:

1. Turing machines and an application of the “crossing-sequences” technique to show a simple lower bound for palindromes.
2. P and NP, Cook’s theorem and NP-completeness, self-reducibility of SAT, and the padding argument to relate, for example, the collapse of P and NP to the collapse of EXPTIME and NEXPTIME.
3. Space complexity, the logarithmic classes L and NL, and Reingold’s recent result showing that  $L = SL$ . Savitch’s theorem and  $PSPACE = NPSPACE$ . The Immerman-Szelepcsényi theorem showing that  $NL = co-NL$ . Interactive proof systems:  $IP = PSPACE$ .
4. Diagonalization and oracles, and the hierarchy theorems. The polynomial time hierarchy (PH), and the linear time hierarchy (LTH), and the Nepomnjascij’s theorem.
5. Circuits, a lower bound for parity (showing that it cannot be computed with bounded depth circuits of polynomial size), and an alternative proof of the same result using the technique of arithmetization. Shannon’s result about boolean circuits, and Razborov’s result showing a lower bound for monotone circuits. Razborov-Smolensky result about circuits with modular gates. Computation with advice. The result of Allender et al showing that  $NL/poly = UL/poly$ .
6. Proof systems, especially resolution, and relation to the DPLL and DP algorithms. Haken’s lower bound for resolution, as well as automatizability and interpolation. Frege systems.
7. Randomized complexity classes: examples of randomized algorithms for perfect matching, primality testing, and pattern matching. The classes RP, co-RP, ZPP, BPP, and counting classes such as PP, #P, and  $\oplus P$ . Chernoff bound and amplification. The isolation lemma and Toda’s theorem.