`Math 3130 Michael Soltys` `Final Exam` `Fall 2007`

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for grader's use	

Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work. ***Show all your work; there will be no credit for answers without a justification.***

Total marks: 100 — ten questions, each worth 10.
Total time: 2 and 1/2 hours.

1. Consider the three vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}, \ \vec{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}.$$

For what values of h is $\vec{y} \in \text{span}\{\vec{v}_1, \vec{v}_2\}$?

2. Consider the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ give by

$$T(x_1, x_2) = (x_1 - 3x_2, 3x_1 + 5x_2, -x_1 + 7x_2).$$

(a) What is the matrix of the linear transformation T?

(b) Is T one-to-one?

(c) Is T onto?

3. (a) Let H, K be vector subspaces of V. Let

$$H \cap K = \{v | v \in H \text{ and } v \in K\}.$$

Show that $H \cap K$ is a vector subspace of V.

(b) If $V = \mathbb{R}^3$ and $H = \text{span}\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\}$, and $K = \text{span}\{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\}$, what is $H \cap K$?

4. Show that the pivot columns of an $n \times m$ matrix A form a basis for $\text{Col}(A)$.

5. Let $S = \{v_1, v_2, \ldots, v_k\}$ be a set of vectors (where $k \geq 1$). Show that S is linearly dependent $\iff$ there exists a vector $v_i \in \text{Span}(S - \{v_i\})$. (Here $S - \{v_i\}$ denotes the set of all the vectors in S but *without* v_i.)

6. Show that for any $m \times n$ matrix A we have that $\dim(\mathrm{Row}(A)) = \dim(\mathrm{Col}(A))$.

7. Let S be a subset of $\mathbb{R}^n$ such that S is finite and contains a non-zero vector. Show that

   (a) $\text{Span}(S)$ is a *subspace* of $\mathbb{R}^n$, and

   (b) a basis for $\text{Span}(S)$ is some *subset* S' of S.

8. Find the eigenvalues of

$$\begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}.$$

Also, find the bases for the corresponding eigenspaces.

9. Show that if A and B are *similar* matrices, then they have the same characteristic polynomial.

10. Show that an $n \times n$ matrix A is *diagonalizable* $\iff$ A has n linearly independent eigenvectors.

End of Exam