**Due on Tuesday June 22 in class**

1. Show that there is a polytime transformation from any RR into a new RR such there is no derived clause in the new refutation which is a *tautology clause* (i.e., a clause which contains both a variable and its negation).

2. Consider the matrix identity $AB = I \rightarrow BA = I$, over the field of two elements (GF(2)). Let $\{[\![AB = I \rightarrow BA = I]\!]_n\}$ be the family of boolean formulas representing this identity, i.e.,

$$\bigwedge_{1\leq i,j\leq n}\left(\left(\bigoplus_{1\leq k\leq n} A_{ik}\wedge B_{kj}\right)\equiv \delta_{i,j}\right)\rightarrow \bigwedge_{1\leq i,j\leq n}\left(\left(\bigoplus_{1\leq k\leq n} B_{ik}\wedge A_{kj}\right)\equiv \delta_{i,j}\right),$$

where $\delta_{i,j}$ is $T$ if $i = j$, and $F$ otherwise. Argue convincingly, but sparingly with details, that extended resolution can prove efficiently this identity.

3. Consider the PPS over the connectives $\{\neg, \vee\}$ (which *is* a complete set of connective), with the following set of rules:

| | |
|---|---|
| $\vdash \neg a \vee a$ | (excluded middle) |
| $a \vdash b \vee a$ | (weakening) |
| $a \vee a \vdash a$ | (contraction) |
| $a \vee (b \vee c) \vdash (a \vee b) \vee c$ | (associative rule) |
| $a \vee b, \neg a \vee c \vdash b \vee c$ | (cut rule) |

   (a) Show that this system is sound and implicationally complete for the set of boolean formulas over the connectives $\{\neg, \vee\}$.

   (b) Consider the following two modifications of this system: (i) allow abbreviations, that is, allow the introduction of new variables $v_{\text{new}}$ and abbreviations $v_{\text{new}} \equiv \phi$, where $\phi$ is any formula not containing $v_{\text{new}}$. And (ii), allow substitutions, that is, given a formula $\phi$, pick any variable in $\phi$, and replace it everywhere in $\phi$ by a new formula $\phi'$. Show that these two variants are $p$-equivalent; the directions (ii) $\Rightarrow$ (i) is easy. For the other direction, it is enough to give an informal justification.