Linear Optimization - Tutorial 1

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About Me

- Carlos Suarez
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- Office Hours: Mondays 4:30pm - 5:20pm
- 4th year Ph.D. student working on research in the area of discrete optimization.
- Hold a M.Eng and a B.Sc from Escuela Superior Politecnica del Litoral.

Linear Optimization Formulation

- $$\begin{aligned} \text{minimize} \quad & c_1 x_1 + \cdots + c_n x_n \\ & a_{11} x_1 + \cdots + a_{1n} x_n = b_1 \\ & \vdots \\ \text{subject to} \quad & a_{m1} x_1 + \cdots + a_{mn} x_n = b_m \\ & x_1 \geq 0, x_2 \geq 0 \ldots x_n \geq 0 \end{aligned} \tag{1}$$

- Can also be rewritten as:

$$\begin{aligned} & \min \ c^T x \\ & \text{s.t. } Ax = b \\ & \text{and } x \geq 0 \end{aligned}$$

Formulations

- For each unit of x_1, there must be at least 3 units of x_2.
  $x_1 \geq 3x_2$ or $x_2 \geq 3x_1$ or...?

- A machine can process either: $11x_1$ kg. per week, or $45x_2$ kg., or $30x_3$kg. per week. What combinations of x_1, x_2 and x_3 can be loaded in 8 weeks?
  .... ≤ 8 or .... ≥ 8 ?

Formulations

- There must be exactly 10 gr. of x_1 for every 15 gr. of x_2.
- Ingredient x_1 must constitute at most 25% of all ingredients X_1, X_2, and X_3?

Example 1

- Hamilton's Poutine is a delivery food business. The company makes a net income of $1.00 for each regular poutine and $1.50 for each deluxe poutine sold. The firm currently has 150 pounds of potatoes and 50 pounds of topping mix. Each regular poutine uses 1 pound of potatoes and 4 ounces (16 ounces= 1 pound) of topping mix. Each deluxe poutine uses 1 pound of potatoes and 8 ounces of topping mix. Based on the past demand per week, Hamilton's Poutine can sell at least 50 regular poutines and at least 25 deluxe poutines. The problem is to determine the number of regular and deluxe poutines they should make to maximize net income. Formulate this problem as an LP problem.

Example 2

- A company makes two products (x_1 and x_2), measured in kg. using two machines (A and B). Each kg. of x_1 that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each kg. of x_2 that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B. At the start of the current week there are 30 kg. of x_1 and 90 kg. of x_2 in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.
  The demand for x_1 in the current week is forecast to be 75 kg. and for x_2 is forecast to be 95 kg. Company policy is to maximise the combined sum of the units of x_1 and the units of x_2 in stock at the end of the week.

Example 3

- Hong Feng makes tables and chairs. Each table can be sold for a profit of $ 30 and each chair for a profit of $10. The carpenter can afford to spend up to 40 hours per week working and takes six hours to make a table and three hours to make a chair. Customer demand requires that he makes at least three times as many chairs as tables. Tables take up four times as much storage space as chairs and there is room for at most four tables each week..

Next Week

- How to solve LP's in Excel using Excel Solver.