Linear Optimization - Tutorial 4

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Week of October 4, 2020

Two phases Simplex Method

$$
\begin{array}{ll}
\text{max} & z = 2x_1 + 3x_2 + x_3 \\
\text{subject to} & x_1 + x_2 + x_3 \leq 40 \\
 & 2x_1 + x_2 - x_3 \geq 10 \\
 & -x_2 + x_3 \geq 10 \\
 & x_j \geq 0 \text{ for } j = 1 \text{ to } 3
\end{array}
$$

Standard Form

Introduce slack variables to convert the problem into standard form:

$$\begin{array}{ll} \text{max} & z = 2x_1 + 3x_2 + x_3 \\ \text{subject to} & x_1 + x_2 + x_3 + s_1 = 40 \\ & 2x_1 + x_2 - x_3 - s_2 = 10 \\ & -x_2 + x_3 - s_3 = 10 \\ & x_j, s_j \geq 0 \text{ for } j = 1 \text{ to } 3 \end{array}$$

Finding a Basic feasible solution

- Simplex method to start needs a initial vertex. In the previous tutorial we used $(0,0,0)$. This time, $(0,0,0)$ does not satisfy the bottom 2 constraints.
- We will form a new problem, introducing artificial variables, to find an initial basic feasible solution(vertex of polyhedron) to our original problem.

Phase I

In order to find a solution(vertex) to start simplex algorithm, consider the artificial minimization problem

$$\text{Min } \sum_{i=1}^{m} v_i \text{ or } \textbf{Max } \sum_{i=1}^{m} -v_i$$
$$\text{s.t. } Ax + v = b$$
$$\text{and } x \geq 0,\ v \geq 0$$

If there is a feasible solution for this problem, it must be $v = 0$. If the original problem has no feasible solution, then $\sum_{i=1}^{m} v_i$ is greater than zero.

Phase I problem

$$
\begin{aligned}
\text{max} \quad & v = -v_1 - v_2 - v_3 \\
\text{subject to} \quad & x_1 + x_2 + x_3 + s_1 + v_1 = 40 \\
& 2x_1 + x_2 - x_3 - s_2 + v_2 = 10 \\
& -x_2 + x_3 - s_3 + v_3 = 10 \\
& x_j, s_j, v_j \geq 0 \text{ for } j = 1 \text{ to } 3
\end{aligned}
$$

Let's continue in Excel, for the iterations of Phase I and II(tableaus).