CS3DB3/SE4DB3/SE6DB3 Tutorial 7

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Outline

Armstrong's Axioms

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X, Y, Z are sets of attributes.

- **Reflexivity:** If $Y \subseteq X$, then $X \rightarrow Y$.
- **Augmentation:** If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z.
- **Transitivity:** If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$.
- **Union:** If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$.
- **Decomposition:** If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$.

Example 1: Proof

Prove the following inference rule for functional dependencies using only Armstrong's axioms:
If $P \rightarrow QR$ and $R \rightarrow S$, then $P \rightarrow QS$.

Solution:

Note: Show the steps of the proof and indicate which of Armstrong's axioms is applied in each step.

$P \rightarrow QR$ (Given)
$P \rightarrow Q$ and $P \rightarrow R$ (Decomposition)
$R \rightarrow S$ (Given)
$P \rightarrow S$ (Transitivity)
$P \rightarrow QS$ (Union)

Example 2: Key/Superkey

Consider the relation $R(A, B, C, D)$ and functional dependencies $\{B \rightarrow C, B \rightarrow D\}$. List all the keys of R.

Solution:
The only key is AB.
$B \rightarrow C$ and $B \rightarrow D$ (Given)
$B \rightarrow CD$ (Union)
$AB \rightarrow ABCD$ (Augmentation with AB)

Exercise: $R(A, B, C, D)$ and functional dependencies $\{AB \rightarrow C, C \rightarrow D, D \rightarrow A\}$. List all the keys of R.