## Examples for the BFGS Quasi-Newton Update

$$
\text { Minimize } f(x)=e^{x_{1}-1}+e^{-x_{2}+1}+\left(x_{1}-x_{2}\right)^{2}
$$

## Iteration 1:

$$
\begin{gathered}
x^{0}=\binom{0}{0} \quad \text { (initial point) } \\
B^{0}=\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) \\
g^{0}=\binom{0.3679}{-2.7183}
\end{gathered}
$$

$s^{0}$ is the solution of $B^{0} s^{0}=-g^{0}$

$$
\begin{gathered}
s^{0}=-B_{0}^{-1} g^{0}=\binom{-0.3679}{2.7183} \\
x^{1}=x^{0}+\alpha_{0} s^{0}
\end{gathered}
$$

Line search with Wolf Condition gives

$$
\begin{gathered}
\alpha_{0}=1 \\
x^{1}=\binom{-0.3679}{2.7183} \\
g^{1}=\binom{-5.9178}{5.9930} \\
\sigma^{0}=x^{1}-x^{0}=\binom{-0.3679}{2.7183} \\
y^{0}=g^{1}-g^{0}=\binom{-6.2856}{8.7113}
\end{gathered}
$$

$$
\begin{gathered}
\triangle B^{0}=\frac{y^{0}\left(y^{0}\right)^{T}}{\left(\sigma^{0}\right)^{T} y^{0}}-\frac{B^{0} \sigma^{0}\left(\sigma^{0}\right)^{T} B^{0}}{\left(\sigma^{0}\right)^{T} B^{0} \sigma^{0}}=\left(\begin{array}{cc}
1.5020 & -1.9737 \\
-1.9737 & 1.9376
\end{array}\right) \\
B^{1}=B^{0}+\triangle B^{0}=\left(\begin{array}{cc}
2.5020 & -1.9737 \\
-1.9737 & 2.9376
\end{array}\right)
\end{gathered}
$$

## Iteration 2:

$$
\begin{gathered}
s^{1} \text { is the solution of } B^{1} s^{1}=-g^{1} \\
s^{1}=-B_{1}^{-1} g^{1}=\binom{1.6082}{-0.9596} \\
x^{2}=x^{1}+\alpha_{1} s^{1},
\end{gathered}
$$

Line search with Wolf Condition gives

$$
\begin{gathered}
\alpha_{1}=1.25 \\
x^{2}=\binom{1.6423}{1.5188} \\
g^{2}=\binom{2.1478}{-0.8422} \\
\sigma^{1}=x^{2}-x^{1}=\binom{2.0102}{-1.1995} \\
y^{1}=g^{2}-g^{1}=\binom{8.0656}{-6.8353} \\
\triangle B^{1}=\frac{y^{1}\left(y^{1}\right)^{T}}{\left(\sigma^{1}\right)^{T} y^{1}}-\frac{B^{1} \sigma^{1}\left(\sigma^{1}\right)^{T} B^{1}}{\left(\sigma^{1}\right)^{T} B^{1} \sigma^{1}}=\left(\begin{array}{cc}
0.3711 & 0.0646 \\
0.0646 & -0.4386
\end{array}\right) \\
B^{2}=B^{1}+\triangle B^{1}=\left(\begin{array}{ccc}
2.8731 & -1.9091 \\
-1.9091 & 2.4990
\end{array}\right)
\end{gathered}
$$

## Iteration 3:

$$
\begin{gathered}
s^{2} \text { is the solution of } B^{2} s^{2}=-g^{2} \\
s^{2}=-B_{2}^{-1} g^{2}=\binom{-1.0635}{-0.4754} \\
x^{3}=x^{2}+\alpha_{2} s^{2},
\end{gathered}
$$

Line search with Wolf Condition gives

$$
\begin{gathered}
\alpha_{2}=1.0313 \\
x^{3}=\binom{0.5456}{1.0285} \\
g^{3}=\binom{-0.3310}{-0.0061} \\
\sigma^{2}=x^{3}-x^{2}=\binom{-1.0967}{-0.4903} \\
y^{2}=g^{3}-g^{2}=\binom{-2.4788}{0.8361} \\
\triangle B^{2}=\frac{y^{2}\left(y^{2}\right)^{T}}{\sigma_{2}^{T} y^{2}}-\frac{B^{2} \sigma_{2} \sigma_{2}^{T} B^{2}}{\sigma_{2}^{T} B^{2} \sigma_{2}}=\left(\begin{array}{cc}
0.2127 & 0.0625 \\
0.0625 & -0.0737
\end{array}\right) \\
B^{3}=B^{2}+\triangle B^{2}=\left(\begin{array}{cc}
3.0858 & -1.8467 \\
-1.8467 & 2.4253
\end{array}\right)
\end{gathered}
$$

Iteration $k \ldots$

