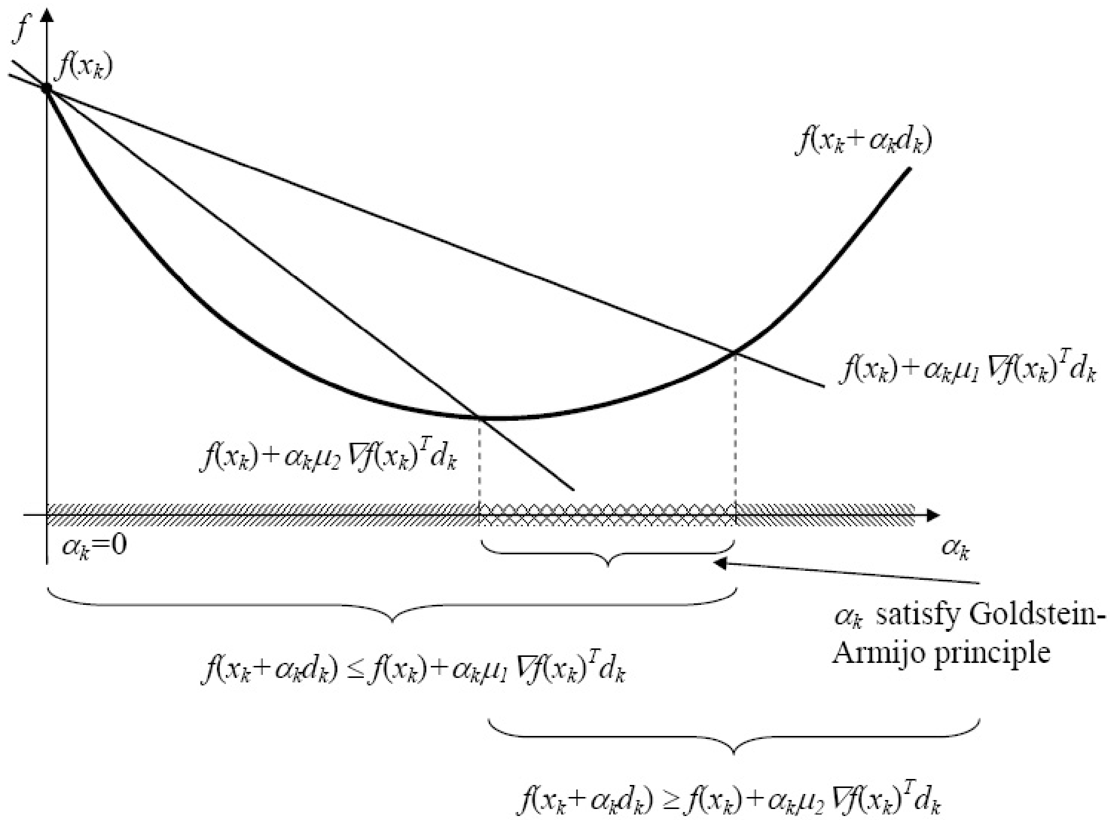


# Goldstein-Armijo line-search

When computing step length $\alpha$ of $f(x_k + \alpha d_k)$, the new point should sufficiently decrease $f$ and ensure that $\alpha$ is away from 0. Thus, we use following bound is used

$$0 < -\alpha_k \mu_1 \nabla f(x_k)^T d_k \leq f(x_k) - f(x_{k+1}) \leq -\alpha_k \mu_2 \nabla f(x_k)^T d_k$$

where $0 < \mu_1 \leq \mu_2 < 1,\ \alpha_k > 0$ and $\nabla f(x_k)^T d_k < 0$. The upper an lower bounds in the above principle ensure $\alpha_k$ is a good choice.

**Algorithm:** Choose parameters $\mu_1$, $\mu_2$, $\rho_1$, $\rho_2$, $\alpha_0$ (for example $\mu_1 = 0.2$, $\mu_2 = 0.8$, $\rho_1 = 1/2$, $\rho_2 = 1.5$, $\alpha_0 = 1$).

**Step 1:** i=0

**Step 2:** `if` $-\alpha_i \mu_1 \nabla f(x_k)^T d_k > f(x_k) - f(x_k + \alpha_i d_k)$

$\Longrightarrow \alpha_{i+1} = \rho_1 \alpha_i$, $i = i + 1$, `goto` **Step 2**

`if` $f(x_k) - f(x_k + \alpha_i d_k) > -\alpha_i \mu_2 \nabla f(x_k)^T d_k$

$\Longrightarrow \alpha_{i+1} = \rho_2 \alpha_i$, $i = i + 1$, `goto` **Step 2**

**Step 3:** $x_{k+1} = x_k + \alpha_i d_k$