ON ASSEMBLE-TO-ORDER SYSTEMS

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Abstract

Since the 1990s, facing increasing competition and mass customization, many companies including Dell have chosen to adopt the assemble-to-order (ATO) model in order to increase products offering and reduce the life cycles of products. Inventory management is a key challenge for ATO systems, in particular determination of inventory replenishment levels without full demand information, component allocations based on available component inventories, and realizations of product demands. ATO systems are usually modeled as a two-stage stochastic integer program. However, such programs are typically hard to solve, especially for stochastic integer nonlinear programs used for the joint optimization. In this thesis, we describe two ATO models proposed by Akçay and Xu (2004) and by Huang (2014). Both models include a nonlinear term in the right hand side of the inventory availability constraints. We discuss the techniques used to linearize the original problem and to estimate the impact of the linearization. In addition, we investigate another key element of ATO systems called component commonality used to reduce inventory costs. An extensive literature review regarding component commonality is provided.

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Notation and abbreviations

t	:	index of periods. Period t is defined as the duration $[t, t+1)$.
i	:	index of components, where $i \in \mathcal{M} = \{1, \cdots, m\}$.
j	:	index of products, where $j \in \mathcal{N} = \{1, \cdots, n\}$.
L_i	:	lead time of component i.
c_i	:	the unit purchasing cost of component i.
L_i	:	lead time of component i.
S_i	:	base stock level of component i.
r_j	:	reward rate of filling a unit demand of product j within response time windo
L	:	$max_{i \in \mathcal{M}} L_i$, maximum lead time.
a_{ij}	:	the number of component i used in each unit of product j.
h_i	:	unit inventory holding cost for component i.
b_j	:	unit backlogging cost for product j.
$P_{j,t}$	:	demand for product j at period t.
$D_{i,t}$	:	demand for component i at period t.
$I_{i,t}$	:	net inventory of component i at the end of period t.
$A_{i,t}$	:	replenishment order of component i arriving at period t.
$(S_i - D_i^{L-i-k})^+$	:	equals to $S_i - D_i^{L_i-k}$ when $S_i \geq D_i^{L_i-k}$;

		equals to 0 when $S_i < D_i^{L-i-k}$.
MTS	:	refer to make-to-stock.
ATO	:	refer to assemble-to-order.
MTO	:	refer to make-to-order.
ETO	:	refer to engineer-to-order.
FCFS	:	refers to first-come, first-served allocation rule.
BOM	:	refers to bills of material.
LP	:	refers to linear program.
RHS	:	refers to the right hand side.
SAA	:	refers to sample average approximation method.

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Chapter 1

Introduction

Samadhi and Hoang [3] classify the production environment as make-to-stock (MTS), assemble-to-order (ATO), make-to-order (MTO), and engineer-to-order (ETO). The classification is based on the concept of customer order. This thesis focuses on ATO systems and, especially, on a two-stage integer program following a first-come, first-served (FCFS) allocation rule.

In this chapter, these four types of production environment and the reasons for selecting ATO systems are discussed first. The outline of the thesis are then addressed.

1.1 Types of production environment

Samadhi and Hoang [3] state that in MTS, standard products, which are relatively predictable, are produced and the end products are inventoried before they are demanded by customers. Since customer orders are fulfilled from the existing inventory, the ability of logistics is much of a concern to the customer, rather than lead time.

Unfortunately, under this environment, demand-supply mismatches may occur. Ordering too much may result in a leftover of inventory. On the other hand, ordering too little may incur the opportunity cost of lost sales [7]. Therefore, inventory planning, lot size determination and demand forecasting are the main operations issues for MTS systems. [26].

In order to reduce the demand-supply mismatches caused by MTS systems, strategies such as ATO, MTO and ETO are developed. When using these strategies, manufacturers attempt to delay producing the end products until they obtain better demand information [7].

In ATO, manufacturers produce components and design bills of material (BOM) structuring from these components. When customer orders arrive, a variety of end products will be assembled using the inventoried components and designed BOM [3]. However, a problem may arise that customer demand must backlogged due to lack of some components, whereby other components remain unused. Determining inventory replenishment levels without full information on product demands and making component allocation decisions depending on available component inventories and realized product demands are main inventory management issues for ATO systems [2].

Unlike ATO, MTO products are typically used to manufacture single-item or small-batch productions and are produced on a unique basis in order to satisfy a customer's specification within the required delivery date [5]. The competitive priority is shorter delivery lead time. Thus the main operations issues are capacity planning, order acceptance or rejection, and attaining high due-date adherence [26].

ETO appears to be an extension of MTO, but its product is designed almost entirely based on customer requirements. The characteristics of each environment are summarized in Table 1.2 [3][30].

Wemmerlöv [30] considers that ATO systems is likely to be a "graduate" stage of either MTS or MTO. A MTS firm, pressured by market, may move to ATO in order to broad the product line and to increase variety of products offered to the market; And a company starting out as a MTO firm may change to ATO such that it might increase demand and reduce delivery time for some of its products. These benefits for ATO systems are implied in Talbe 1.1.

Since the 1990s, increased competition and more demanding customers have forced companies to seek ways to provide a large product variety specified by the customers and shorten delivery lead times without increasing cost. One of successful cases is Dell Computers who applies ATO systems to a subset of its products. Traditional computer companies give the customer few combinations of options and have lead times on the order of weeks or months due to backlogged orders [24]. By allowing the customer to select among processors, monitors, disk drives, etc, Dell satisfies the needs and wants of the customer and thus gains advantages on high volumes, low cost, speedy delivery and large variety of products. The success of Dell has attracted most other companies in the personal computer market to adopt similar ATO systems.

As we see, ATO systems play an important role in the industry and, in particular, in the electronics industry and the automobile industry. In the last decade, more and more researchers become to focus on this topic.

Characteristics	MTS	ATO	MTO	ETO
Product	Standard	Defined product family	No typical product family, customized	Fully customized
Product demand	Can be forecast	$\Longleftrightarrow$		Cannot be forecast
Customer delivery time	Shortest	$\Longleftrightarrow$		Longest
Production lead time	Unimportant to customer	Important	Important	Most important
Key Competitiveness	Logistics	Final assembly	Fabrication, final assembly	Whole process
Width of product line	Medium	High	Low	Low
Production volume of each sales unit	Highest	$\Longleftrightarrow$		Lowest
Complexity of operation	Distribution	Assembly	Component manufacturing	Engineering
Uncertainty of operation	Lowest	$\Longleftrightarrow$		Highest
Handling of demand uncertainty	Safety stocks of sales units	Overplanning of components and subassemblies	Little uncertainty exists	Little uncertainty exists
Interface between production function and customer	Lowest	$\Longleftrightarrow$		Highest
Bill of Material Structuring	Standard BOMs (one BOM for each sales item)	Planning BOMs are used	BOMs are unique and created for each customer order	BOMs are unique and created for each customer orde

Table 1.2: Characteristics of MTS, ATO, MTO and ETO

1.2 Preliminary

1.2.1 Review period

The review period can be classified into three types, namely single period models, periodic review models and continuous review models.

As the simplest of all inventory models, **single period models** have the essential characteristic of making only a single procurement during a single time period of finite length [13]. The major difficulty for single period models is to forecast the demand since at the end of the period, because the unsold items becomes obsolete [19]. The typical problem for this type of model is the news-vendor problem, a problem of deciding how many newspapers should be purchased on a given day. If the demand exceeds the supply of papers, lost sales may cause unsatisfied customers to look elsewhere for newspaper in the future, while if the demand is less than the quantity ordered, the news-vendor will suffer the cost of disposing of the day old paper, which have no value [28].

In **periodic review models**, the inventory level is examined only at discrete, and usually constant intervals, e.g., at the end of each week, and decisions, such as whether or not to place an order, are made only at these review times [14]. Some simple component allocation heuristics, such as the Product Based Priority (PBP)allocation policy [31], the Fair Share (FS) [1] allocation policy and the FCFS [17] allocation policy, are proposed in this type of model.

In **continuous review models**, an order is placed as soon as the inventory level drops to the prescribed reorder point [14]. The traditional method of implementing a continuous review model is to use a two-bin system. In more recent years, computerized inventory systems, which record each addition to inventory and each sale causing a withdrawal electronically, have largely replace two-bin systems. The modern scanning devices, usually seen at retail store checkout, aim to adjust the current inventory levels.

1.2.2 Types of inventory models

The **demand** for a product in inventory is the number of units that will need to be withdrawn from inventory for some use, like sales, during a specific period [14]. Based on the predictability of demand involved, inventory models are usually divided into two categories, namely deterministic models and stochastic models. A **deterministic** inventory model would be used when the demand in future period is known so that it can be forecast with considerable precision. However, when the future demand is considerable uncertainty, it is necessary to use a **stochastic** model where the demand in any period is a random variable having a known probability distribution, i.e., Possion distribution, normal distribution, uniform distribution, etc.

1.2.3 Types of demand for a product

Independent demand does not depend on the demand for any of the company's other products and its model is also sold separately from other models.

When the product is just one component that is used to assembled into the company's end product, e.g., television speakers and television sets. the demand for this component will depends on the demand for the end product. Such demand is so-called **dependent demand** [14].

1.2.4 Linear programs

A **linear function** is a function of the form $c_1x_1 + \cdots + c_nx_n$, where $x_1, \cdots, x_n$ are decision variables, and $c_1, \cdots, c_n$ are given constants called the coefficients of the variables in the function. A **linear program** (LP) is an optimization problem in which all the constraint functions and the objective function are linear functions. The constraint is either an equality constraint, in the form $g_i(x) = b_i$ or an inequality constraint, in the form $g_i(x) \geq b_i$ or $g_i(x) \leq b_i$. The given function of the decision variables, $g_(x)$, is called the constraint function and the given constant, b_i, is called the right hand side (RHS) constant. If the constraint function involves only one variable, such as $x_j \geq b_j$ or $x_j \leq b_j$, such constraint is called bound constraint or bound, lower or upper, on individual variables.

A vector $x = (x_1, \cdots, x_n)^T$ is called a feasible solution for the problem if it satisfies all the constraints. A feasible solution having the best value for the objective function among all the feasible solutions is called optimum solution [22].

1.2.5 Two-stage model

In a **two-stage model**, the first stage consists in deciding the optimal base stock of various components, and the second stage consists in deciding component allocation based on on-hand inventory and realized demands.

Suppose we have the following problem:

$$\max \quad \mathbb{E}_\xi \left[Q\left(S, \xi(\omega)\right)\right] \tag{1.1}$$

$$s.t. \quad AS_i \leq b, \quad S_i \geq 0, \tag{1.2}$$

where $Q(S, \xi(\omega))$ is the optimal value of the seconde-stage problem

$$\max \quad q(\omega)^T x \tag{1.3}$$

$$s.t. \quad Wx \leq h(\omega) - T(\omega)S_i, \quad x \geq 0. \tag{1.4}$$

Here, the decision in the first stage is $S = (S_1, \cdots, S_m)$. In the second stage, a number of random vectors ξ may realize with the second-stage problem data $q(\omega), h(\omega), T(\omega)$ and W. For each realization, the second-stage decision x is taken, but typically, the decisions x are not the same under different realizations of ξ. The objective function of Equation 1.1 is the expectation of the second-stage objective $q(\omega)^T x$ taken over all realizations of the random vector ξ [14] [6].

1.2.6 Sample average approximation (SAA) method

Consider the following stochastic programming problem:

$$\max \quad \mathbb{E}_\xi \left[Q(S, \xi) \right] \tag{1.5}$$

Note that in the framework of two-stage stochastic programming, the objective function $Q(S, \xi)$ in Equation 1.5 is given by the optimal value of the corresponding second-stage problem.

A random sample $\xi^1, \cdots, \xi^N$ of N realizations the the random vector ξ is generated in computer by using Monte Carlo sampling techniques or is viewed as historical data of N observations of ξ. Then the expected objective function $Q(S, \xi)$ is estimated by averaging values $Q(S, \xi^j)$, where $j = 1, \cdots, N$. This is the so-called **sample average approximation** (SAA) method. Therefore, the SAA problem of 1.5 can be written as follow:

$$\max \quad \left\{ \frac{1}{N} \sum_{j=1}^{N} Q(x, \xi^j) \right\}. \tag{1.6}$$

The process is repeated with different samples to obtain candidate solutions.

This technique is used to solve stochastic optimization problems, and particularly, is suitable stochastic problems have a huge set of random variables and using exact mathematical programming techniques is not effective [29] [25][2].

1.2.7 Multi-matching

Multi-matching is a matching between supply of multiple components and demand of multiple products. Multi-matching is between multiple units of supply and demand, instead of one unit. For example, for a given component i, at period t, a demand of $D_{i,t} = \sum_{j=1}^{n} a_{ij} P_{j,t}$ units of component i is realized. Then we regard the $\sum_{j=1}^{n} a_{ij} P_{j,t}$ units of components i as a whole and satisfy this demand with a supply of $\sum_{j=1}^{n} a_{ij} P_{j,t}$ units of component i. Therefore, a multi-matching between supply and demand is established. Furthermore, we extend such matching to all components.

1.2.8 Other basic definitions

In **FCFS** allocation policy, customer orders of a particular period cannot be allocated by the system until all the earlier orders are satisfied.

When implementing a **base-stock policy**, also known as the order-up-to level, inventory is ordered to keep inventory position, i.e., on-hand inventory plus on-order inventory minus backorders) equaling the base stock level [7].

1.3 Thesis outline

In chapter 2, we describe two examples of ATO systems where a stochastic program is established for jointly optimizing the base stock levels and component allocation. In chapter 3, we introduce component commonality, one of key elements in ATO systems, and provide an extensive literature review regarding component commonality. Finally, we discuss some opportunities for future work in chapter 4.

Chapter 2

Examples of removing the absolute value in the right-hand-side of the inventory availability constraints

In order to jointly optimize the base stock levels and component allocation, in general, a two-stage stochastic integer program is established. However, such program is extremely hard to solve, and especially for the stochastic integer nonlinear programs. In the following, we describe two ATO models proposed by Akçay and Xu [2] and by Huang [17]. Both models consist a nonlinear term in the RHS of the available on-hand inventory constraints. We discuss how the authors attempt to remove these nonlinear terms and estimate the impact of the linearization.

2.1 Akçay and Xu's model

2.1.1 ATO system setting

The ATO model proposed by Akçay and Xu [2] has the following basic assumptions:

- The system is under periodic review.
- For each component, an independent base stock (order-up-to level) policy is used.
- A FCFS allocation rule is used to satisfy demands for different periods.
- Product demands in each periods are random variables.
- The replenishment lead time for each component is deterministic and can be different for different components.
- A reward is received if the order is fulfilled within given period length from the arrival of demand, referred to as response time window.

In addition, in each period, the sequence of events, illustrated in Figure 2.1, is assumed as following: inventory position of each component is reviewed (IPR) $\rightarrow$ new replenishment orders of components are placed (NOP) $\rightarrow$ earlier replenishment orders arrive (ROA) $\rightarrow$ demands are realized (DR) $\rightarrow$ components are allocated and products are assembled (CAPA) $\rightarrow$ associated reward is accounted (ARA). Note that the final assembly times are negligible.

Now, we derive serval equations that will aid to formulate the model. Let $D_i[s,t]$ and $A_i[s,t]$ represent the total demand and total replenishment of component $i, i =$

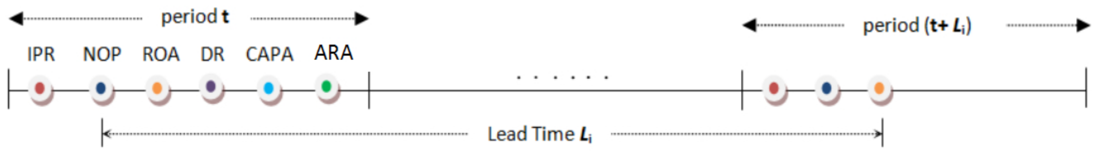

Figure 2.1: Sequence of events

$1, 2, \cdots, m$ from period s through period t, respectively. Then we have $D_i[s, t] = \sum_{k=s}^{t} D_{i,k}$ and $A_i[s, t] = \sum_{k=s}^{t} A_{i,k}$, for $i = 1, \cdots, m$, where $0 \le k \le L_i$.

The net inventory of component i at the end of period t under the base-stock control S_i is as follows:

$$I_{i,t} = S_i - D_i[t - L_i, t], \quad i = 1, \cdots, m. \tag{2.7}$$

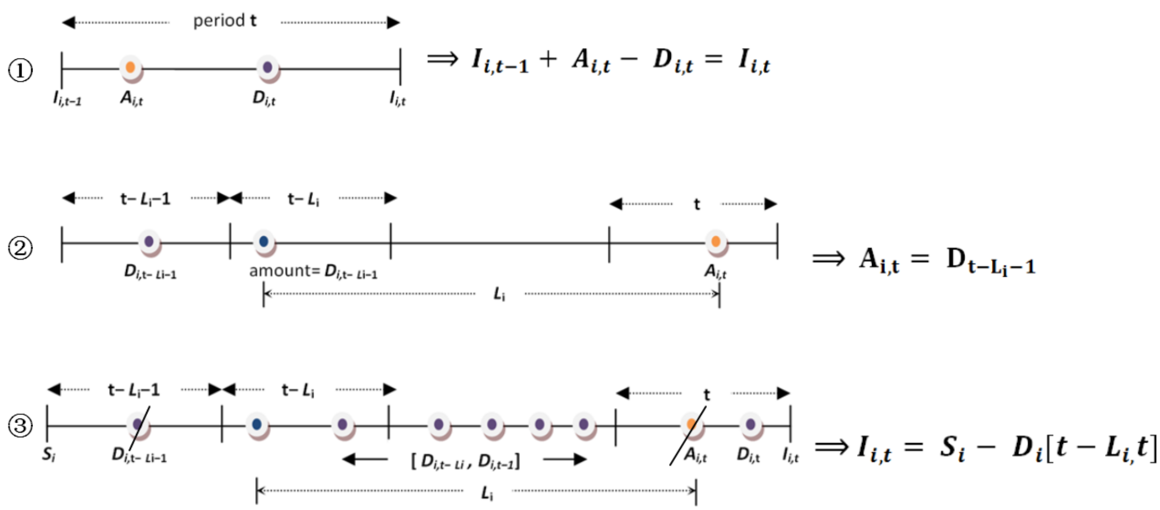

Figure 2.2: Detailed Steps for deriving Equation 2.1

In Equation 2.1, we replace t by $t + k$, then the net inventory of component i at the end of period $t + k$ is represented as

$$I_{i,t+k} = S_i - D_i[t + k - L_i, t + k], \quad i = 1, \cdots, m. \tag{2.8}$$

Since FCFS allocation policy is assumed to use in the system, thus the inventory of component i at the end of period $t+k$ (i.e., the inventory of component i at the end of period $t-1$ plus total replenishment of component i from period t through period $t+k$ minus the total demand from period t through period $t+k$), is given by

$$I_{i,t+k} = I_{i,t-1} + A_i[t, t+k] - D_i[t, t+k]. \tag{2.9}$$

Substituting Equation 2.3 into Equation 2.2, we reach the following equation:

$$I_{i,t-1} + A_i[t, t+k] - D_i[t, t+k] = S_i - D_i[t+k-L_i, t+k]. \tag{2.10}$$

Rearranging Equation 2.4 and using the identity $D_i[t+k-L_i, t-1] = D_i[t, t+k] - D_i[t+k-L_i, t+k]$, which is explained in Figure 2.3, we get

$$I_{i,t-1} + A_i[t, t+k] = S_i + D_i[t+k-L_i, t-1]. \tag{2.11}$$

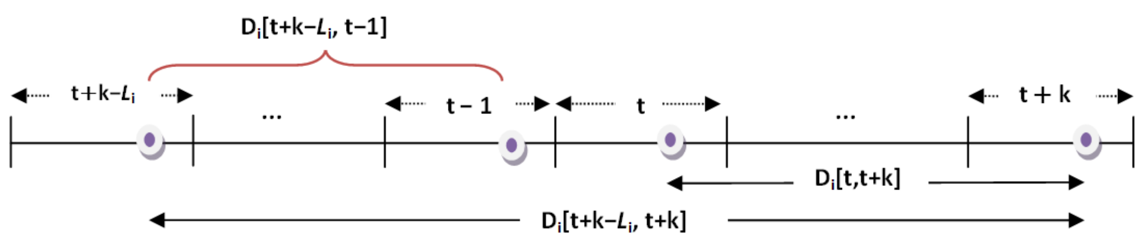

Figure 2.3: $D_i[t+k-L_i, t-1] = D_i[t, t+k] - D_i[t+k-L_i, t+k]$

$I_{i,t-1} + A_i[t, t+k]$, on the left hand side of Equation 2.4, is the net inventory of component i in period $t+k$ after receiving all replenishment orders from periods t through $t+k$, but before fulfilling any order received after period $t-1$. Therefore, the available on-hand inventory of component i at period $t+k$ that can be used for filling the demand at period t, $P_{1,t}, \cdots, P_{n,t}$ is defined as

$$(S_i - D_i[t+k-L_i, t-1])^+ = max\{S_i - D_i[t+k-L_i, t-1], 0\}. \tag{2.12}$$

The index of periods t can be dropped in steady state and hence $D_{i,t}$ and $P_{j,t}$ are replaced by D_i and P_j. The stationary version of $D_i[t+k-L_i, t-1]$ can also be simplified as $D_i^{L_i-k}$, the total stationary demand of component i in $L_i - k$ periods. In addition, the lead time for component i is assumed to be greater or equal to the response time window of product j, i.e., $L_i \geq w_j$ for all i and j. This assumption is reasonable because if $L_i < w_j$, then a product j requesting component i can be fulfilled before its response time window so that it is not necessary to make the allocation decision of component i for the product j order.

2.1.2 Two-stage stochastic integer programming formulation

The ATO system is formulated as a two-stage stochastic integer program. The sample average approximation (SAA) method is used to solve the first-stage, subject to an inventory investment budget constraint, and a problem of maximizing the total reward of filled orders within their response time windows is developed in the second-stage.

In the first stage, we determine the base-stock levels , which are constrained by a pre-set budget B. After making the base-stock-level decisions, we know the customer orders of various products. Suppose that we have a collection of M random sample demands of N realizations of random vector ξ. Let $\xi_l^N = (\xi(\omega_l^1), \xi(\omega_l^2), \cdots, \xi(\omega_l^N))$ be the realizations of l-th sample, where $l = 1, \cdots, M$. Then for each sample l, realization h, we maximize the total reward attainable from the orders $P_1, P_2, \cdots, P_n$ within the response time windows $W = \{\omega_1, \omega_2, \cdots, \omega_n\}$, denoted by $Q_W(S_l, \xi(\omega_l^h))$.

$$Q_W(S_l, \xi(\omega_l^h)) = \max \left\{ \sum_{j=1}^{n} \sum_{k=0}^{\omega_j} r_j x_{jk}^h \right\} \tag{2.13}$$

$$s.t. \quad \sum_{k=0}^{\omega} x_{jk}^h \leq P_j^h \quad j = 1, \cdots, n \tag{2.14}$$

$$\sum_{j=1}^{n} \sum_{u=0}^{k} a_{ij} x_{ju}^h \leq (S_i - D_i^{L_i - k})^+ \quad i = 1, \cdots, m, \quad k = 0, \cdots, \omega \tag{2.15}$$

$$x_{jk}^h \geq 0 \; and \; integer \quad j = 1, \cdots, n, \quad k = 0, \cdots, \omega. \tag{2.16}$$

Note that $\omega_1 \leq \omega_2 \leq \cdots \leq \omega_n = \omega$.

Let $Q_W^N(S_l)$ be the expected value of $Q_W(S_l, \xi_l^h)$, for $h = 1, \cdots, N$. Since the SAA method is implemented, instead of calculating the expectation of objective function with $Q_W(S) = \mathbb{E}_\xi[Q_W(S, \xi)]$, we calculate the average of the objective function, which is shown as following:

$$Q_W^N(S_l) = \frac{1}{N} \sum_{h=1}^{N} \sum_{j=1}^{n} \sum_{k=0}^{\omega_j} r_j x_{jk}^h. \tag{2.17}$$

Finally, the two-stage stochastic integer programming formulation of the ATO system is as follow:

$$\max_{S_l} \left\{ Q_W^N(S_l) = \frac{1}{N} \sum_{h=1}^{N} \sum_{j=1}^{n} \sum_{k=0}^{\omega_j} r_j x_{jk}^h \right\}. \tag{2.18}$$

$$s.t. \quad \sum_{i=1}^{m} c_i S_i \leq B \tag{2.19}$$

$$\sum_{k=0}^{\omega} x_{jk}^h \leq P_j^h \quad j = 1, \cdots, n, \quad h = 1, \cdots, N \tag{2.20}$$

$$\sum_{j=1}^{n} \sum_{u=0}^{k} a_{ij} x_{ju}^h \leq (S_i - D_i^{L_i - k})^+ \quad i = 1, \cdots, m, \quad k = 0, \cdots, \omega, \quad h = 1, \cdots, N \tag{2.21}$$

$$x_{jk}^h \geq 0 \; and \; integer \quad j = 1, \cdots, n, \quad k = 0, \cdots, \omega, \quad h = 1, \cdots, N \tag{2.22}$$

$$S_i \geq 0 \text{ and integer} \quad i = 1, 2, \cdots, m. \tag{2.23}$$

The decision variable x_{jk}^h is in the h-th realization, the number of customer orders for product j that are filled k periods after they are received, for $0 \leq k \leq \omega$. Objective function (2.13) requires that rewards for the orders filled are not collected after their response times windows. Constraint (2.14) requires that the total units of product j assembled within its response time window do not exceed the demand of product j, $j = 1, \cdots, n$. Constraint (2.15) requires that the components i used for assembly within the first k periods, $0 \leq k \leq \omega$, cannot exceed the available on-hand inventory of component i, for $i = 1, 2, \cdots, m$.

2.1.3 Upper bound and lower bound

Let $Q_W^N(\widehat{S}_l)$ be the optimal value of the problem formulated in section 2.1.2, where $\widehat{S}_l$ is the optimal base-stock-level vector, where $l = 1, \cdots, M$. Then the average of these M optimal values, $\overline{Q}_W^N$, is given by

$$\overline{Q}_W^N = \frac{1}{M} \sum_{l=1}^{M} Q_W^N(\widehat{S}_l). \tag{2.24}$$

Generate a different sample $\xi^{N'}$ of size N', where $N' \gg N$, then we have

$$Q_W^{N'}(\widehat{S}_l) = \frac{1}{N'} \sum_{h=1}^{N'} Q_W(\widehat{S}_l, \xi_0^h), \tag{2.25}$$

where $Q_W(\widehat{S}_l, \xi_0^h)$ is the optimal solution of the second-stage problem with the base-stock-level vector $\widehat{S}_l$ and realized demand ξ^h, for $h = 1, \cdots, N'$. Therefore, for each sample, we solve the following optimization problem:

$$\max_{\widehat{S}_l} \left\{ Q_W^{N'}(\widehat{S}_l) = \frac{1}{N'} \sum_{h=1}^{N} \sum_{j=1}^{n} \sum_{k=0}^{\omega_j} r_j x_{jk}^h \right\}, \tag{2.26}$$

$$s.t. \quad \sum_{i=1}^{m} c_i S_i \leq B \tag{2.27}$$

$$\sum_{k=0}^{\omega} x_{jk}^h \leq P_j^h \quad j = 1, \cdots, n, \quad h = 1, \cdots, N' \tag{2.28}$$

$$\sum_{j=1}^{n} \sum_{u=0}^{k} a_{ij} x_{ju}^h \leq (S_i - D_i^{L_i - k})^+ \quad i = 1, \cdots, m, \quad k = 0, \cdots, \omega, \quad h = 1, \cdots, N' \tag{2.29}$$

$$x_{jk}^h \geq 0 \textit{ and integer} \quad j = 1, \cdots, n, \quad k = 0, \cdots, \omega, \quad h = 1, \cdots, N' \tag{2.30}$$

$$S_i \geq 0 \textit{ and integer} \quad i = 1, 2, \cdots, m. \tag{2.31}$$

Let $\widehat{Q}_W^{N'}(\widehat{S}_l)$ be the optimal objective function of the above problem, $l = 1, 2, \cdots, M$. Then the "best" solution among the M candidate solutions, $\widehat{S}^*$, is defined as

$$\widehat{S}^* \in \arg\max\{\widehat{Q}_W^{N'}(\widehat{S}_l), l = 1, \cdots, M\}. \tag{2.32}$$

Let $Q_W(S^*, \xi)$ be the optimal solution of the problem for any possible realization, given by

$$Q_W(S_l, \xi(\omega_l^h)) = \max \left\{ \sum_{j=1}^{n} \sum_{k=0}^{\omega_j} r_j x_{jk}^h \right\} \tag{2.33}$$

$$s.t. \quad \sum_{k=0}^{\omega} x_{jk}^h \leq P_j^h \quad j = 1, \cdots, n \tag{2.34}$$

$$\sum_{j=1}^{n} \sum_{u=0}^{k} a_{ij} x_{ju}^h \leq (S_i - D_i^{L_i - k})^+ \quad i = 1, \cdots, m, \quad k = 0, \cdots, \omega \tag{2.35}$$

$$x_{jk}^h \geq 0 \textit{ and integer} \quad j = 1, \cdots, n, \quad k = 0, \cdots, \omega, \tag{2.36}$$

where S^* is the vector of optimal base-stock levels. It is well known that $\widehat{Q}_W^{N'}(\widehat{S}^*) \leq Q_W(S^*,\xi) \leq \overline{Q}_W^N$. Therefore, $\widehat{Q}_W^{N'}(\widehat{S}^*)$ is the lower bound of $Q_W(S^*,\xi)$ and $\overline{Q}_W^N$ is the upper bound of $Q_W(S^*,\xi)$.

2.1.4 Impact of linearization

The problem given in (2.18)-(2.22) is nonlinear since there exists a nonlinear term $(S_i - D_i^{L_i-k})^+$ in the available on-hand inventory Constraint (2.21). We observe that in order to speed up the computation, Akçay and Xu [2] remove the plus sign in the Constraint (2.21), replacing this nonlinear term with $S_i - D_i^{L_i-k}$, in their computation experiments. Unfortunately, if the plus sign is ignored, the original problem is totally different by changing the feasible region and the upper bound.

When the given budge is very small, the base-stock level for component i, S_i, is so low that it can not fill the demand of component i in $L_i - k$ period. Thus the value of $S_i - D_i^{L_i-k}$ is negative. If the plus sign is kept, then the value of $(S_i - D_i^{L_i-k})^+$ becomes to 0, and there exists a feasible solution $x_{ju}^h = 0$. The left hand side of Constraint 2.15 is required to be nonnegative, but while the plus sign is removed, it is upper bounded by a negative value. It is impossible to find a solution for the latter problem. As a result, one impact of removing the absolute value in the right hand side of the constraint 2.15 is that some of the feasible solutions become infeasible.

When the plus sign is dropped, the reduction of the feasible solutions causes the value of $\overline{Q}_W^N$ in Equation 2.21 to become smaller. This leads to the decrease of the upper bound of $Q_W(S^*,\xi)$.

Both Akçay and Xu [2] and Deza et al. [8] choose Zhang's [31] system, consisted of four products and five components, to develop their computational experiments.

The problem setting of Zhang's system is described in Table 2.3.

				Component				
				i	1	2	3	4	5
				c_i	2	3	6	4	1
Product				L_i	3	1	2	4	4
j	$Mean$	$StdDev$	r_j	Bill of Material					
1	100	25	1		1	2	1	0	0
2	150	30	1		1	1	1	0	0
3	50	15	1		0	1	1	1	0
4	30	11	1		0	0	0	1	1

Table 2.3: Problem setting of Zhang's system

In the following, we attempt to analyze the impact to real world by comparing the experiment results shown by Akçay and Xu [2] and Deza et al. [8]. Recall that Akçay and Xu [2] remove the absolute value in the RHS of the inventory availability constraint while Deza et al. keep it when solving the problem computationally. Table 2.4 lists the upper bounds provided in both papers for given budget between $\$8,000-\$10,000$.

B	UB in Akçay and Xu	UB in Deza et al.
$8,000$	70.54	75.01
$9,000$	88.85	90.02
$10,000$	98.12	98.34
$11,000$	99.66	99.86

Table 2.4: Upper bounds computed by Akçay and Xu and by Deza et al.

Suppose that a manufacturer adopts an ATO system same to Zhang's system and attempts to obtain a reward without exceeding 90. Akçay and Xu's computational results suggest the manufacturer to invest more than $\$9,000$, but Deza et al. argue that even if he invests exact $\$9,000$, he will get a reward larger than 90. Therefore, although Akçay and Xu have an advantage in saving computing time and reduce computing cost, sometimes their results would result in manufacturer making wrong decisions.

Furthermore, we observe that the difference between the upper bounds, showed in Table 2.4, decreases when the given budget increases. In order to reduce the effect caused by removing plus sign, Akçay and Xu start with a budget of $\$8,000$. However, Deza et al. show that the given budget could start at $\$2,000$, listed in Table 2.5.

Budget	C_1	C_2	C_3	C_4	C_5	LB	UB
2000	0	0	0	428	199	9.08	9.11
3000	0	0	162	413	376	9.08	9.12
4000	0	325	249	339	175	9.45	9.88
5000	613	492	383	0	0	21.59	22.98
6000	699	598	468	0	0	46.47	47.83
7000	782	722	545	0	0	65.78	66.49
7500	819	786	584	0	0	71.73	71.98
8000	865	846	622	0	0	74.88	75.01
8500	766	727	562	316	151	81.13	82.40

Table 2.5: Experiment results for budget between $\$2,000$ to $\$8,500$

Suppose a manufacturer is new in ATO system and does not want to invest lots of money at the beginning. In this case, he could not get information about reward under low budget from Akçay and Xu's experiment results so that he is hard to make a decision on whether to invest or not.

2.2 Huang's model

2.2.1 ATO system setting

In Huang's [17] model, the basic assumptions for the ATO system are as following:

- The system is under periodic review.

- For each component, an independent base stock (order-up-to level) policy is used.
- A FCFS allocation rule is used to satisfy demands for different periods.
- Product demands in each periods are random variables.
- The replenishment lead time for each component is deterministic and can be different for different components.

The sequence of events in each period is assumed as following: inventory position of each component is reviewed (IPR) $\rightarrow$ new replenishment orders of components are placed (NOP) $\rightarrow$ earlier replenishment orders arrive (ROA) $\rightarrow$ demands are realized (DR) $\rightarrow$ components are allocated and products are assembled (CAPA) $\rightarrow$ inventory holding cost and backlogging cost accounted. Note that the final assembly times are negligible.

The sequence of events clearly shows that unlike Akçay and Xu's [2] model, Huang addresses a cost-minimization optimization model.

2.2.2 Two-stage stochastic integer programming

The concept of multi-matching is applied int this model so that only exactly $\sum\limits_{j=1}^{n} a_{ij}P_j$ units of supply to its corresponding demand, no more, and no less. Therefore, by period $t+k$, the total number of supply that could be used to satisfy the demand in period t is:

$$O_i^k = \min\left\{(S_i - D_i^{L_i-k})^+, \sum_{j=1}^{n} a_{ij}P_j\right\}. \tag{2.37}$$

Note that $O_i^{-1} = 0, O_i^{L_i} = \min\{S_i, \sum_{j=1}^{n} a_{ij}P_j\}$, and $O_i^{L_i+1} = \min\{S_i + \sum_{j=1}^{n} a_{ij}P_j, \sum_{j=1}^{n} a_{ij}P_j\} = \sum_{j=1}^{n} a_{ij}P_j$. Equation 2.25 means that if the available on-hand inventory for component i is large enough to fill the demand in period t, then suppose exactly $\sum_{j=1}^{n} a_{ij}P_j$ units of component i. Otherwise, supply all the available on-hand inventory of component i we have. O_i^k is a piecewise linear nonconvex functions of S_i.

There are three types of cost involved in this models. The first type is called remnant stock holding cost, which incurs when the demand D_i at period t is so large that it can not be fulfilled until period $t + L_i + 1$. By period $t + k$, there are $\sum_{\mu=0}^{k} \sum_{j=1}^{n} a_{ij}x_{j\mu}$ units assembled into end products, where $x_{j\mu}$ is decision variable, representing the assembled end product j by period $t + k$ for the demand arriving at period t. Then the value of the remnant stock at period $t+k$ is $O_i^k - \sum_{\mu=0}^{k} \sum_{j=1}^{n} a_{ij}x_{j\mu}$. Therefore, the remnant stock holding cost at period $t + k$ is given by

$$h_i(O_i^k - \sum_{\mu=0}^{k} \sum_{j=1}^{n} a_{ij}x_{j\mu}). \tag{2.38}$$

The second type is the order backlogging cost. The order backlogging incurred at period $t + k$ that used to satisfy the demand at period t is:

$$b_j(P_j - \sum_{mu=0}^{k} x_{j\mu}). \tag{2.39}$$

The third type, so-called the classical inventory holding cost, incurs when the demand D_i is so small that the available component stock at period t could satisfy this demand at current period and the left inventory will be carried to the next period and used for future demands. The classical inventory holding cost at period t

is counted as

$$h_i[(S_i - D_i^{L_i-0})^+ - \sum_{j=1}^{n} a_{ij} P_j]^+. \tag{2.40}$$

In the following, a two-stage stochastic integer program is modeled as follows:

$$\min \mathbb{E}_\xi [C(S, \xi(\omega)] \tag{2.41}$$

$$s.t. \quad S_i \in \mathbb{Z}_+ \qquad \forall i \in \mathcal{M}, \tag{2.42}$$

where

$$\begin{aligned} C(S, \xi(\omega)) = \min \Bigg\{ & \sum_{i=1}^{m} h_i \left[(S_i - D_i^{L_i-0})^+ - \sum_{j=1}^{n} a_{ij} P_j \right]^+ \\ & + \sum_{i=1}^{m} \sum_{k=0}^{L+1} h_i (O_i^k - \sum_{\mu=0}^{k} \sum_{j=1}^{n} a_{ij} x_{j\mu}) + \sum_{j=1}^{n} \sum_{k=0}^{L+1} b_j (P_j - \sum_{\mu=0}^{k} x_{j\mu}) \Bigg\} \end{aligned} \tag{2.43}$$

$$s.t. \quad \sum_{k=0}^{L+1} x_{jk} = P_j \qquad \forall j \in \mathcal{N} \tag{2.44}$$

$$\sum_{\mu=0}^{k} \sum_{j=1}^{n} a_{ij} x_{j\mu} \le O_i^k \qquad \forall i \in \mathcal{M}, k \in \mathcal{L} \tag{2.45}$$

$$x_{jk} \in \mathbb{Z}_+ \qquad \forall j \in \mathcal{N}, k \in \mathcal{L}. \tag{2.46}$$

Observe from Constraint (2.32) that all the demand of product j in period t should be satisfied no later than period $t + L + 1$. Constraint (2.33) states that all the components used for assembly come from the available inventory.

2.2.3 Linearization techniques

The above problem is nonlinear since there are nonlinear items in both the objective function (2.31) and Constraint (2.33). Here in order to compare with Akçay and Xu's model, we only discuss how Huang linearizes the Constraint (2.13).

Since O_i^k is defined as $\min\{(S_i - D_i^{L-i-k})^+, \sum_{j=1}^{n} a_{ij}P_j\}$, the Constraint (2.33) can be written as the following two new constraints:

$$\sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} \leq (S_i - D_i^{L_i-k})^+ \tag{2.47}$$

$$\sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} \leq \sum_{j=1}^{n} a_{ij}P_j. \tag{2.48}$$

Observe that Constraint (2.35) is automatically guaranteed by the constraint (2.32). Therefore, the constraint (2.33) can be replaced by the constraint (2.35), but there still exists a nonlinear item $(S_i - D_i^{L_i-k})^+$. Now Huang faces the same problem to Akçay and Xu, instead of directly dropping the plus sign, Huang uses a standard linearization techniques, introducing an additional binary variable, to linearize $(S_i - D_i^{L_i-k})^+$.

Then we introduce the "big-M" notation, and replace Constraint (2.35) by:

$$\begin{aligned}
\sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} &\leq Mz_{ik} \\
\sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} &\leq S_i - D_i^{L_i-k} + M(1 - z_{ik}) \\
S_i - D_i^{L_i-k} &\leq Mz_{ik} \\
z_{ik} &\in 0, 1,
\end{aligned} \tag{2.49}$$

where M is a big positive number.

This technique works because if $S_i \geq D_i^{L_i-k}$, then the value of z_{ik} is set to be 1. We have

$$\begin{aligned} \sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} &\leq M \\ \sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} &\leq S_i - D_i^{L_i-k} + 0 \\ S_i - D_i^{L_i-k} &\leq M. \end{aligned} \tag{2.50}$$

Furthermore, Equation 2.38 can be simplified to be $\sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} \leq S_i - D_i^{L_i-k}$.

If $S_i \leq D_i^{L_i-k}$, then set the value of z_{ik} as 0. Thus the Constraint (2.35) is modified as

$$\begin{aligned} \sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} &\leq 0 \\ \sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} &\leq S_i - D_i^{L_i-k} + M \\ S_i - D_i^{L_i-k} &\leq 0. \end{aligned} \tag{2.51}$$

Since M is a big positive number, the value of $S_i - D_i^{L_i-k} + M$ is greater than 0. As a result, we get $\sum_{\mu=0}^{k}\sum_{j=1}^{n} a_{ij}x_{j\mu} = 0$.

Chapter 3

Component commonality

3.1 Introduction to component commonality

Component commonality is defined as the use of the same version of a component across multiple products [20]. Component commonality has been widely recognized as a key element in ATO systems, aiming to reduce the cost of safety stock.

It is well-known that the power supply in Europe is 220V and that in America is 110V. Suppose a firm sells printers both in Europe and in the United States. Instead of producing two unique components, 110V power supply and 220V power supply, the manufacturer designs a universal power supply to replace these two unique components. Using the universal component, the firm does not need to forecast demand for both printers to determine how many of each type of power supplies should be produce since the shortage of power supplies for one type of printers could be met by the surplus of power supplies for the other type of printers. This effect is so-called risking pooling, which allows the firm to maintain smaller inventory levels of the common component [16]. Usually the common component is more expensive than the unique one. Therefore, the cost of the common component must be considered as an important factor when determining whether or not to use the common component.

3.2 Problem

Fully component commonality is used in Zhang's system [31]. We illustrate this model in Figure 3.4.

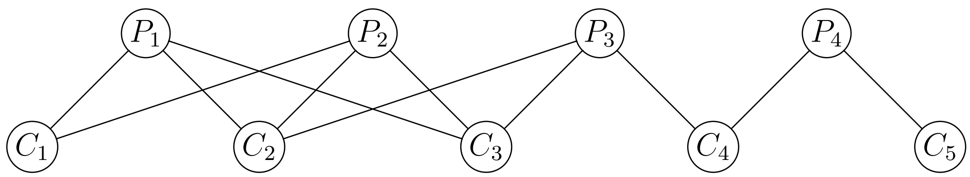

Figure 3.4: Bill of Material with fully component commonality

Deza et al. [8] find that when the given budge is quite low, the optimal base stock levels for some components are zeros, as shown in Table 2.5 . For example, when the budget is $\$2,000$, the optimal base stock levels for components C_1, C_2, and C_3 are set to zeros, which implies that only product 4 are assembled. Since product 4 is assembled by one unit of component 4 and one unit of component 5, the total reward for product 4 is 199 when the budge is $\$2,000$.

It is easy to observe that if we separate the inventory of component 4 with respect to products 3 and 4, as illustrated in Figure 3.5, we may get a better reward.

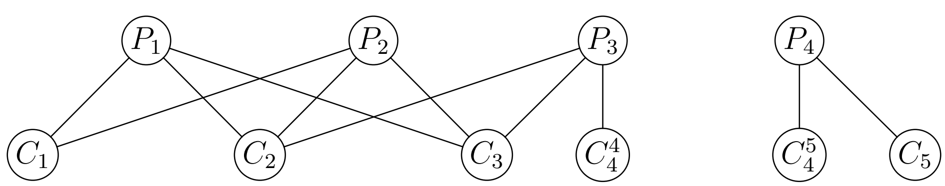

Figure 3.5: Bill of Material with partial component commonality

When the budget is \$2,000, we invest all the money to buy component C_4^5 and component C_5, so that we can get 400 units for each. Therefore, the reward is 400, which is much larger than 199.

We can get an ATO model without component commonality when separating inventories with respect to different products. Deza et al. conclude that when the give budge is low, i.e., $\$2,000 - \$8,000$, component commonality is not beneficial.

It is an appealing idea to find the conditions when component commonality is beneficial by comparing the case with commonality and the case without commonality in different types of review period. In the following, we summarize the studies focused on the area of component commonality.

3.3 Literature review

Much research has been done to show that utilizing commonality is beneficial when it is possible for single period models. Baker et al. [4] study the effect of commonality on optimal safety stock level by establishing a specialized inventory model which consists of two end products, each comprising two different components, one of which is common to both products. They consider the problem of minimizing safety stock level while satisfying a service level constraint under independent uniform demand distributions. Having compared the case with commonality and the case without commonality algebraically, they find that the introduction of commonality induces a reduction in the optimal safety stock level due to a large decrease in the optimal stock of the common component and a slight increase in the optimal stock of the product-specified components. Gerchak et al. [12] extend this work by investigating whether the properties identified in Baker et al. [4] hold for a model consisted arbitrary number of products under joint demand distributions. By solving a problem that minimizes the total stocking cost while meeting a service-level constraint, they find that when it is beneficial to introduce commonality, the service-level measure that is used plays an important role on the change of inventory level. When the bottleneck measure is used, every specialized component's inventory level is nondecreasing with the introduction of commonality; however, for the aggregate measure, when the costs of product-specific components are not equal, the sum of the specialized components' stock levels may decrease when taking advantage of commonality. And when the costs of product-specific components are equal, the stock levels of the common components decrease and those of the product-specific components increase.

Both Baker et al. [4] and Gerchak et al. [12] assume the costs of the product-specified component and the replacing (common) component to be identical; However, it is more realistic to allow the common component to be more expensive than those it replaces. Eynan and Rosenblatt [9] present three models to compare and analyze the effects of increasing component commonality and demonstrate that some forms of commonality may not always be advisable to use. The non-commonality(basic) model's configuration is comprised of two products, each having two different components, and the demand for each product follows a uniform distribution. The two commonality models' configurations are with one of the components being common to the two end products and with both of the components being common to the two end products respectively. By formulating a problem of minimizing the total component purchasing cost subjected to service-level constraints and solving the optimal inventory level of the various components, they provide conditions for which commonality should be either employed or avoided, bounds on the total saving that maybe obtained by using commonality, and changes in inventory levels resulting from changing configuration. Mirchandani and Mishra [21] compare a non-commonality model with two products under the assumption of uniform demand distribution to two different commonality models depending on whether the products are prioritized or nonprioritized. They use Baker et al. [4]'s configurations and develop cost-minimizing optimization models under product-specific service-level constraints. They compare their results with Baker et al. [4] and Gerchak et al. [12], which use the aggregate service level measure as well, in order to check whether or not the properties indicated in those paper hold for the three models considered in this paper. They include a parameter called usage to allow the products require arbitrary units of each component.

In all the work we have previously mentioned, results are investigated by comparing non-commonality model and commonality model. Non-commonality configuration and commonality configuration used by Baker et al. [4] are appreciated for multi period model as well. Hillier [15] develops a simple multiple-period model (periodic-review system with zero lead times) having uniformly distributed demands and derives a closed-form optimal (or near-optimal) solution by solving a purchase costs minimization model with service level constraint. The results show that in a multiple-period model, if the common component does not exceed the price of the ones it replaces, then it is always worthwhile to employ the common component; However, if the common component is more costly than the ones it replace, then it almost never worthwhile to employ the common component, which is drastically different from the cases in single period. Hillier [16] extends this to an any number of final products, any number of components model. Song and Zhao [27] formulate a continuous-review system with cost minimization and Poisson distributed demand for one common component, two-product system. This study indicates that if component commonality is generally worthwhile to be used, its value depends strongly on the component costs, lead times, and allocation rules (first-in-first-out and modified first-in-first-out). For example, when lead times are identical and modified first-in-first-out allocation rule is used, component commonality always guarantees inventory benefits. And the results also that modified first-in-first-out always outperforms first-in-first-out ont the value of commonality.

An idea, utilized by Jönsson and Silver [18] and Fong et al. [10], is to allocate a given budge among the components. Jönsson [18] consider the problem of maximizing the expected total of end items sold when demands for end products follow independently normal distribution. They analyze the impact of commonality among components for a specific model consisted of two end products, each with two components, one of which is common to both products by developing a heuristic allocation procedure. Fong et al. [10] use Baker et al. [4]'s configuration and provide analytical results for a commonality problem that minimizes the expected units shortage subjected to a fixed budged constraint under independent Erlang demand distributions. They observe that the relative reduction is in the expected units shortage can be substantial when the budget level is high relative to the demand requirements for the end products even if the component component is much more expensive.

Unfortunately, both of these work operate for single period. In this paper, extension of this idea to periodic-review system for a profit-maximization model conducted by Gerchak and Henig [11] and Nonås [23]. Gerchak and Henig [11] develop an any number of products with arbitrary components for a single period to discuss the qualitative implications of component commonality. They show that the optimal stock level of product-specific components always increase when commonality is employed. Nonås [23] outlines a gradient based search procedure to find the optimal inventory level for large product system by dealing with the profit maximization commonality problem for a three products and any number of common components system and a general distributed demand in a multi period.

A simple comparison among these studies is listed in Table 3.7.

Author \ Item	Review Period	Demand Distribution	Optimization Problem	BOM	Constraint
Baker et al.(1986)	single	uniform distribution	min safety stock level	two products, each with two components	service level
Gerchak et al.(1988)	single	joint distribution	min total stocking cost	arbitrary number of products	service level
Eynan and Rosennblatt(1996)	single	uniform distribution	min purchasing cost	two products, each with two components	service level
Mirchandani and Mishra(2002)	single	uniform distribution	min purchasing cost	two products, each with two components	service level
Hillier(1999)	multiple	uniform distribution	min purchasing cost	two products, each with two components	service level
Hillier(2000)	multiple	uniform distribution	min purchasing cost	any number of products and components	service level
Song and Zhao(2009)	continuous	Poisson distribution	min purchasing cost	two products, each with two components	service level
Jönsson and Silver(1989)	single	normal distribution	max the expected total end items sold	two products, each with two components	service level
Fong et al.(2004)	single	Erlang distribution	min the expected units shortage	two products, each with two components	given budget
Gerchak and Henig(1986)	single	Erlang distribution	max profit	any number of products and components	given budget
Nonås(2009)	multiple	general distribution	max profit	three products and with any number of components	given budget

Table 3.7: Comparison of literature review

Chapter 4

Conclusion

ATO systems are increasingly important in modern economy as it could satisfy the customer's requirement with great production variety and short life cycles of products. Usually a two-stage stochastic integer program is modeled to find the optimal base stock level and component allocation. However, even for a simple model, such as four products and five components, it might be challenging to solve. Therefore, researchers not only focus on how to model ATO systems but also try to approximate the model using techniques such as sample average approximation and linearization technique discussed in this thesis.

We describe two examples proposed by Akçay and Xu [2] and Huang [17] to analyze techniques used in their work. We find that Huang [17] uses equivalent linearization to safely remove the absolute value in the RHS of the inventory availability constraint while keeping the feasible solutions unchanged. Akçay and Xu first use the sample average approximation method to estimate the expected objective functions, and then remove the absolute value in the RHS of the inventory availability constraint in order to speed up the computation. Compared with Akçay and Xu's model, Huang's model requires more time but the computational results are more trustable. However, although we argue that the feasible region and upper bound would be changed when directly dropping plus sign. We use the computation results shown in Deza et al. [8] to point out when the given budget is large, the impact of directly dropping the plus sign is quite small and thus one can give up accuracy in exchange of increasing computational speed and reducing computational cost.

My main contribution is in chapter 3. Deza et al. [8] remarked that, when the given budget is low, avoiding component commonality is beneficial in contrast with most. This intriguing observation motivates my future research dealing with component commonality. I surveyed previous approaches and classify them based on period review, objective function of optimization problem, satisfied constraint, etc. and a number of insights are presented. For example, the literature suggests that it is more realistic to allow the common component to be more expensive than those unique component it replaces. This idea should be incorporated in our current model. The most commonly used model used in the literature consists of two products and three components, with one of them is common to both products. However, even such simple model can be extremely hard to analysis with periodic review.

In my future research, I plan to tackle a reversed Λ system, see Figure 4.6, and consider the cost of common component as a key variable to understand the benefit and drawbacks of component commonality.

And then I will attempt to extend the Λ model to a more complex model, which is shown in Figure 4.7.

Figure 4.6: Component commonality and Non-component commonality for reversed Λ system

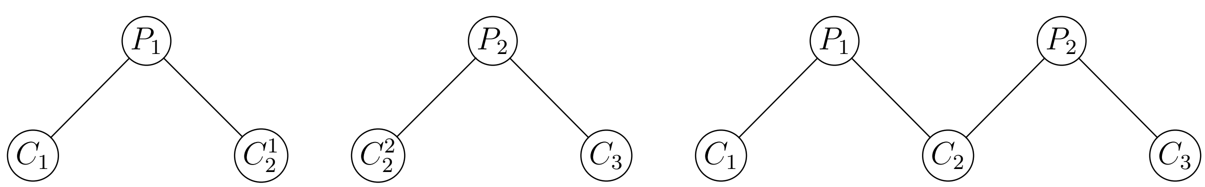

Figure 4.7: Component commonality and Non-component commonality for a 2-product, 3-component system

Bibliography

[1] N. Agrawal and M.A. Cohen. Optimal material control in an assembly system with component commonality. *Naval Research Logistics*, 48(5):409–429, 2001.

[2] Y. Akçay and S.H. Xu. Joint inventory replenishment and component allocation optimization in an assemble-to-order system. *Management Science*, 50(1):99–116, 2004.

[3] T.M.A. Ari Samadhi and K. Hoang. Shared computer-integrated manufacutring for various types of production environment. *International Journal of Operations & Production Management*, 15(5):95–108, 1995.

[4] K.R. Baker, M.J. Magazine, and H.L.W. Nuttle. The effect of commonality on safety stock in a simple inventory model. *Management Science*, 32(8):982–988, 1986.

[5] F. Ballestin, C. Schwindt, and J. Zimmermann. Resource leveling in make-to-order production: modeling and heuristic solution method. *International Journal of Operations Research*, 4(1):50–62, 2007.

[6] J.R. Birge and F. Louveaux. *Introduction to stochastic programming.* Springer, New York, 2nd edition, 2011.

[7] G. Cachon and C. Terwiesch. *Matching supply with demand: an introduction to operations management*. McGraw-Hill/Irwin, New York, NY, 2006.

[8] A. Deza, K. Huang, H. Liang, and X.J. Wang. On component commonality for a periodic review assemble-to-order system. AdvOL-Report.

[9] A. Eynan and M.J. Rosenblatt. Component commonality effects on inventory costs. *IIE Transactions*, 28(2):93–104, 1996.

[10] D.K.H. Fong, H. Fu, and Z. Li. Efficiency in shortage reduction when using a more expensive common component. *Computers & Operations Research*, 31(1):123–138, 2004.

[11] Y. Gerchak and M. Henig. An inventory model with component commonality. *Operations research letters*, 5(3):157–160, 1986.

[12] Y. Gerchak, M.J. Magazine, and A.B. Gamble. Component commonality with service level reqirements. *Management Science*, 34(6):753–760, 1988.

[13] G. Hadley and T.M. Whitin. *Analysis of inventory systems*. Prentice-Hall, Englewood Cliffs, NJ, 1963.

[14] F.S. Hillier and G.J. Lieberman. *Introduction to operations research*. McGraw-Hill, New York, NY, 9th edition, 2010.

[15] M.S. Hillier. Component commonality in a multiple-period inventory model with service level constraints. *International Journal of Production Research*, 37(12):2665–2683, 1999.

[16] M.S. Hillier. Component commonality in multiple-period, assemble-to-order systems. *IIE Transactions*, 32(8):755–766, 2000.

[17] K. Huang. Cost minimization in a periodic review assemble-to-order system. In press, 2014.

[18] H. Jönsson and E.A. Silver. Optimal and heuristic solutions for a simple common component inventory problem. *Engineering Costs and Production Economics*, 16(4):257–267, 1989.

[19] C. Kao and W.K. Hsu. A single-period inventory model with fuzzy demand. *Computers & Mathematics with Application*, 43(6-7):841–848, 2002.

[20] E. Labro. The cost effect of component commonality: a literature review through a management accounting lens. *Manufacturing & Service Operations Management*, 6(4):358–367, 2004.

[21] P. Mirchandani and A.K. Mishra. Component commonality: Models with product-specific service constraints. *Production and Operations Management*, 11(2):199–215, 2002.

[22] K.G. Murty. *Operations research: deterministic optimization models.* Prentice-Hall, Englewood Cliffs, NJ, 1995.

[23] S.L. Nonås. Finding and identifying optimal inventory levels for systems with common components. *European Journal of Operational Research*, 193(1):98–119, 2009.

[24] Y.K. Ro, J.K. Liker, and S.K. Fixson. Modularity as a strategy for supply

chain coordiantion: the case of U.S. auto. *IEEE Transactions on Engineering Management*, 54(1):172–189, 2007.

[25] A. Shapiro, D. Dentcheva, and A. Ruszczyński. *Lectures on stochastic programming: modeling and theory.* MPS-SIAM, Philadelphia, PA, 2009.

[26] C.A. Soman, D.P. Van Donk, and G. Gaalman. Combined make-to-order and make-to-stock in a food production system. *Internaltional Journal of Production Economics*, 90:223–235, 2004.

[27] J.S. Song and Y. Zhao. The value of component commonality in a dynamic inventory system with lead times. *Manufacturing & Service Operations Management*, 11(3):493–508, 2009.

[28] R.J. Tersine. *Principles of inventory and materials management.* North-Holland, New York, 1988.

[29] B. Verweij, S. Ahmed, A.J. Kleywegt, G. Nemhauser, and A. Shapiro. The sample average approximation method applied to stochastic rounting problems: a computational study. *Computational Optimization and Applications*, 24(2-3):289–333, 2003.

[30] U. Wemmerlöv. Assemble-to-order manufacturing: implications for materials management. *Journal of Operations Management*, 4(4):347–368, 1984.

[31] A.X. Zhang. Demand fulfillment rates in an assemble-to-order system with multiple products and dependent demands. *Production and Operations Management*, 6(3):309–324, 1997.