Linear Optimization - Tutorial 2

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Excel Solver

- Excel has a built in solver module that can solve Linear Programs (and IPs/NLPs)
- This tutorial will show how to enable the solver add-in and how to use it to solve a LP

Load the Solver Add-in in Excel

- https://support.office.com/en-us/article/load-the-solver-add-in-in-excel-612926fc-d53b-46b4-872c-e24772f078ca
- Windows:
  - File − > Options − > Add-ins − > Manage − > Excel Add-ins
  - Select Solver Add-in, then OK.
- Mac:
  - Tools − > Excel Add-ins
  - Select Solver Add-in, then OK.
- Solver button is available in the Data tab.

Example 1 - Scheduling Postal Workers

Each postal worker works for 5 consecutive days, followed by 2 days off, repeated weekly.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Demand	17	13	15	19	14	16	11

Minimize the number of postal workers. (We allow fractional workers on each day).

Formulating the Linear Program

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Demand	17	13	15	19	14	16	11

Formulating the Linear Program

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Demand	17	13	15	19	14	16	11

$$
\begin{aligned}
\text{min} \quad & z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \\
\text{subject to} \quad & x_1 + x_4 + x_5 + x_6 + x_7 \geq 17 \\
& x_1 + x_2 + x_5 + x_6 + x_7 \geq 13 \\
& x_1 + x_2 + x_3 + x_6 + x_7 \geq 15 \\
& x_1 + x_2 + x_3 + x_4 + x_7 \geq 19 \\
& x_1 + x_2 + x_3 + x_4 + x_5 \geq 14 \\
& x_2 + x_3 + x_4 + x_5 + x_6 \geq 16 \\
& x_3 + x_4 + x_5 + x_6 + x_7 \geq 11 \\
& x_j \geq 0 \text{ for } j = 1 \text{ to } 7
\end{aligned}
$$

Assigning decision variables in Excel

- Pick cells to represent each of the decision variables.
- These may be left blank, but make a note of which cell is for which decision variable.
- In the demonstration, I have chosen cells C2 to I2 to represent x1 – x7.
- These cell numbers will be entered into the solver and given values when a solution is found.
- You may find it helpful for organization to place all of these variables in a line, with labels in adjacent cells.

Assigning the objective function in Excel

- Pick a cell to represent the objective function.
- I’ve chosen D6, and given it the value “=C2+D2+E2+F2+G2+H2+I2” (minus the quotations) to represent z=x1+x2+x3+x4+x5+x6+x7 in terms of the decision variable cells.
- Note that it is the equals sign at the start which tells Excel it is an equation.
- If no values have been given to C2-I2, D6 will now appear as 0, and will change based on updates to those values.

Assigning the constraints in Excel

- Pick cells to represent each of the LHSs of the constraints.
- The equations for these must be entered into these cells in terms of the decision variable cells. For example, to represent "$x_1+x_4+x_5+x_6+x_7$," I've entered "=C2+F2+G2+H2+I2" into cell D10.
- For organization, I've placed these equations in cells D10-D16, and typed the equations they represent in the adjacent cells to serve as labels.

Assigning the constraints in Excel, cont.

- Pick cells to represent each of the RHSs of the constraints.
- As these are just numbers, simply type the value into a cell.
- For organization, I have chosen to enter the RHSs into cells F10-F16, and entered ≥ symbols as labels into cells E10-E16, so that they clearly match up with the corresponding LHS of the constraint.

Opening Solver

- By default, the Solver tool is not enabled.
- Go to Options, click the “Add-ins” tab, select “Excel Add-ins” in the “Manage” drop down, and click “Go.”
- Check “Solver Add-in” and click OK.
- Solver can now be found in the Data tab in the Analysis toolbox.

Using Solver

- Opening Solver, we get a form to tell Excel what we're trying to optimize and what the variables and conditions are.
- "Set Objective" refers to the objective function. Enter the cell reference of the objective function (in the example, I would enter D6).
- Excel seems to default to "absolute references" here, which appears as D6 instead of D6, but unless you're moving cells around at this stage, either form of reference will work.

Using Solver, cont.

- “To” is asking for how the problem is to be optimized. We will always want either a minimum or maximum. In this case, set it to “Min”.
- “By Changing Variable Cells” requires a list of all the decision variables. As I have listed them in a row, I can simply type C2:I2 to reference all 7 decision variables.

Using Solver, cont.

- “Subject to the Constraints” needs us to list the constraints for the problem.
- Click “Add” to open a form to add constraints.
- For “Cell Reference”, reference the LHS cell
- Our inequalities are all of the form LHS ≥RHS, so select >= from the drop down
- For “Constraint”, reference the corresponding RHS cell.
- You could also just type the RHS into the “Constraint” box, but this way I can list all my constraints in one line (D10:D16 >= F10:F16).

Using Solver, cont.

- As we’re dealing with an IP problem in this case, we also need to constrain our variables to integer values.
- To do this, add another constraint, reference all of the decision variable cells, and select “Int” from the drop down.
- We should also check “Make Unconstrained Variables Non-Negative” to implement the $x_i \geq 0$ constraints.

Last steps

- As the objective function and constraints are linear, any of the solving methods should provide the correct result. I have selected Simplex LP, as this is the method that we will learn to solve by hand in the coming weeks.
- Pressing "Solve," we obtain the solution:
  x_1=6, x_2=3, x_3=3, x_4=7, x_5=0, x_6=3, x_7=1, z=23
- Note that this solution is not unique, so a different solving method may find different x_i values, but z will always total 23.
- If you need more clarification or examples for how to use the Excel solver tool, a tutorial can be found on the MIT opencourseware at: http://ocw.mit.edu/courses/sloan-school-of-management/15-053-optimization-methods-in-management-science-spring-2013/tutorials/MIT15_053S13_tut03.pdf

Next Week

- Assignment Questions
- Simplex Method