Linear Optimization - Tutorial 3

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Example 1

- A local 24-hour grocery store has the following shift requirements for its employees:

Shift	1	2	3	4	5	6
Time period	4-8	8-12	12-16	16-20	20-24	24-4
Employees required	6	25	13	17	9	6

Shift 1 follows immediately after shift 6. An employee works eight hours consecutively, starting at the beginning of one of the six shifts. The owner wants to find the least number of employees required in order to operate the store.
Formulate the problem as a linear program (LP).

Example 1

Setting $x_i(i = 1, 2, , 6)$to be the number of employees beginning work at the start of period i, we can model this problem as follows:

$$
\begin{aligned}
& & \min \quad & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \\
\text{s.t.} & & & x_1 + x_6 \geq 6 \\
& & & x_1 + x_2 \geq 25 \\
& & & x_2 + x_3 \geq 13 \\
& & & x_3 + x_4 \geq 17 \\
& & & x_4 + x_5 \geq 9 \\
& & & x_5 + x_6 \geq 6 \\
& & & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{aligned}
$$

This linear programming system is an integer program.

Example 2

A furniture manufacturer has 4000 units of Walnut, 5000 units of Maple, and 6000 units of Oak in stock. It can create 3 products using these raw materials, with input requirements as given below. A table must always be sold with 4 chairs and a desk must be sold with a chair. Chairs can be sold individually. You are asked to find the product mix (i.e. how many tables, desks and chairs to produce respectively) that maximizes the revenue.

	Wood needed (unit/item)			Profit
	Walnut	Maple	Oak	Revenue/item
Table	25	50	90	$100
Desk	15	30	50	$50
Chair	10	10	25	$10

Example 2

1. Formulate the problem as an Linear Program (LP)
2. Solve the linear program to find the maximum revenue (and how many of each to sell)? *(A software tool may be used but a screenshot of the result must be provided)*

Example 2

1. Formulate the problem as an Linear Program (LP)
Let x_1 be the number of tables created.
Let x_2 be the number of desks created.
and x_3 be the number of chairs created.

$$\begin{aligned} & \max 100x_1 + 50x_2 + 10x_3 \\ s.t. \quad & 25x_1 + 15x_2 + 10x_3 \leq 4000 \\ & 50x_1 + 30x_2 + 10x_3 \leq 5000 \\ & 90x_1 + 50x_2 + 25x_3 \leq 6000 \\ & 4x_1 + x_2 \leq x_3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Example 2

2. Solve the linear program to find the maximum revenue (and how many of each to sell)? *(A software tool may be used but a screenshot of the result must be provided)*
Maximum Revenue: $4,800 from 80 desks and 80 chairs.