3EA3 Lecture Notes

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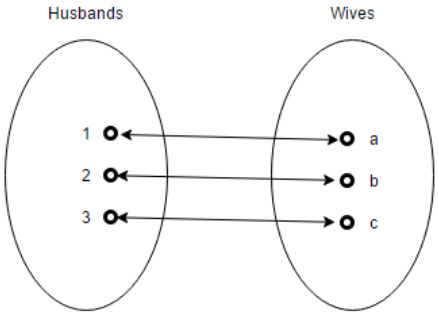

Galois Connections

Bijection

- One-to-one correspondence/isomorphism
- Everything can be paired up
- Example: Husbands and wives

$f : A \cong B$

$\equiv$

$\exists g : B \to A \quad \cdot \quad f;g = id = g;f$

Equal means: $h = k$

$$\forall x \cdot h(x) = k(x)$$

(L, U) Galois Connection

$\equiv$

$$\forall x, y \cdot Lx \sqsubseteq y \equiv x \sqsubseteq' Uy$$

$$(C, \sqsubseteq') - L \to (S, \sqsubseteq)$$

$$(C, \sqsubseteq') \leftarrow U - (S, \sqsubseteq)$$

C (complex) domain switch to S (simpler) domain using Galois connection.

Example: Integer Division

$x \div y :=$ greatest integer division below x/y

$= \left\lfloor \frac{x}{y} \right\rfloor$

$$\forall k \cdot \quad k \quad \leq \quad x \div y \quad \equiv \quad k \times y \quad \leq \quad x$$

$U = \div y$
$L = \times y$

$\Rightarrow$ {Take k to be $x \div y$}

$$x \div y \quad \leq \quad x \div y \quad \equiv \quad (x \div y) \times y \quad \leq \quad x$$

$\equiv$ {$\leq$ is reflexive: $z \leq z \equiv true$;
$true$ is identity of $\equiv$: $true \equiv p \equiv p$}

$$(x \div y) \times y \quad \leq \quad x$$

Example:

$$1 \quad \leq \quad x \div x$$

$$\forall k \cdot \quad k \quad \leq \quad x \div y \quad \equiv \quad k \times y \quad \leq \quad x$$

$$\Rightarrow \{\text{Take } k, y := 1, x\}$$

$$1 \quad \leq \quad x \div x \quad \equiv \quad 1 \times x \quad \leq \quad x$$

$$\equiv \{\leq \text{ is reflexive: } z \leq z \equiv true; \\ 1 \text{ is identity of } x : 1 \times z = z\}$$

$$1 \quad \leq \quad x \div x$$

Try:

$$a \div (b \div c)$$

$$= \quad \leq \quad \geq$$

$$(a \times c) \div b$$