CompSci3EA3 - Lecture Notes Feb 8th

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-Read the textbook! :) it will help with the quizzes
-For this, read chapter 9

1 Function Patching

$for \quad f : X \to Y$
$\quad\quad e : X$
$\quad\quad E : Y$
let
$f[e \mapsto E](x) =$
$\quad\quad$ if $\quad x = e$
$\quad\quad$ then $\quad E$
$\quad\quad$ else $\quad f(x)$
$\quad\quad fi$

Example:
$f(x) = x^2 + 3$
$f[0 \mapsto 12](x) =$
$\quad\quad$ if $\quad x = 0$
$\quad\quad$ then $\quad 12$
$\quad\quad$ else $\quad f(x)$
$\quad\quad fi$

$f[0 \mapsto 12][3 \mapsto 5] =$
$\quad\quad$ if $\quad x = 3$
$\quad\quad$ then $\quad 5$

else if $x = 0$ then 12
else $f(x)$
fi

2 Wrapping up Assignments

"b[i] := e" :
- b at index i becomes e

array:
- is a sequence of memory
- essentially a function

Array: $b[0..N-1]$ over type T $\approx$ function $b : 0..N-1 \to T$

In accordance with Function Patching, the definition of an array assignment is as follows:

$$\text{"b[i] := e"} = \text{"b := b[i} \mapsto e\text{]"}$$

Definition *

$$wp\text{"}b[i] := e\text{"}\ R$$

= ?

Q: How would you represent this with the assignment rule?
R[b[i] := e] ? – **Incorrect**

Why?
- b[i] is not a variable!

Remember the assignment rule?? :
Assignment Rule :

$$wp\ \text{"}x := E\text{"}\ R\ =\ R[x := e]$$

$(\wedge E\ well\ defined\ \wedge\ E\ has\ same\ type\ as\ x)$
- where x is a **variable**, and R & E are **terms**

Correct Answer:
R[b $:=$ b[i $\mapsto$e]]$(e\ well{-}defined,\ has\ type\ of\ elem\ in\ b,\ i\ valid\ index\ of\ b)$

Example :

Given $b[0..N-1]$ *of type* $\mathbb{N}$, *we calculate* :

wp " $i\ :=\ 5;\ b[i] = 5/x$"R

= { sequence with wp }

wp " $i\ :=\ 5$"$(wp$ " $b[i]\ =\ 5/x$"$R)$

= { Definition * $\wedge$ x $\in \mathbb{N}\wedge$ no-div-by-zero }

wp " $i\ :=\ 5$"$(R[b := b[i \mapsto 5/\text{x}]\ ]" \wedge 0 \leq i < N \wedge 0 \leq \wedge\ \text{x} \neq 0)$

= {Assignment axiom $\wedge$ Substitution over function application }

$(R[b\ :=\ b\ [i \mapsto 5/x]\ ]\ )[i := 5] \wedge (0 \leq i < N)[i := 5] \wedge (0 < x)[i := 5]$

= {Textual substitution }

$R[b\ :=\ b[5 \mapsto 5/x]\ ] \wedge 0 \leq 5 < N \wedge 0 < x$

3 Guards and Weakest Preconditions

Condition 1
Def:
wp if

$\square\ B_0 \to S_0$
$\square\ B_1 \to S_1$
...
$\square\ B_{n-1} \to S_{n-1}$
$fi \quad R$
$\equiv$
$(\exists i : 0..N-1 \cdot B_i)\ \wedge$ // one of the guards have to be true
$(\ \forall i : 0..N-1 \cdot wp\ S_i R)$ // chosen guards statement give you R

Theorem:

$$wp\ \text{“if } A \to X\ \square\ B \to Y\ fi\text{ ”}\ R$$
$$\equiv (A\ \vee\ B) \wedge (A \Rightarrow\ wp\ X\ R) \wedge (B \Rightarrow\ wp\ Y\ R)$$

t : “$x + 5 - 2 * z$”
free variables: x, z

**** Terms correspond to functions of their free vars! ****

$f(x, z) = t$
$= x + 5 - 2 * z$

Def:
State = function vars $\to$values

Theorem

function application distribute over conditional

f if b then E else F fi
= if b then $f(E)$ else $f(F)$ fi

****Note** ‘=’ is equality here, not an arbitrary equivalence relation**

Why does it fail if it’s E and F are programs?