Computer Algebra vs Computer Analysis

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July 8, 2004
Overview

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• Systems and UI issues will also be ignored
Definition: Computer Algebra

Abstract domains: rings, fields, semi groups, categories, etc but also $M_{n \times n} [\mathbb{Z}_p [\alpha_1, \ldots, \alpha_m]][[x]]$
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Algebraic algorithms: arithmetic for all the above objects, term-rewriting, etc

• Gröbner bases
• Matrix factorizations
• Elimination in non-commutative Ore algebras
• Differential Algebra
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Computational Exact Classical Analysis

- The study of functions
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Key idea 1: finite representations of functions.
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Computational Exact Classical Analysis

- The study of functions

Key idea 1: finite representations of functions.

Key idea 2: functions are not necessarily computable!
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- Parametric problems are always assumed “generic”
- These are not bugs! They are non-trivial theoretical issues.
Hands-on!

Specific examples of problems, using Maple.

But these are common to all Computer Algebra systems.