Readers and Writers

CS 3SD3

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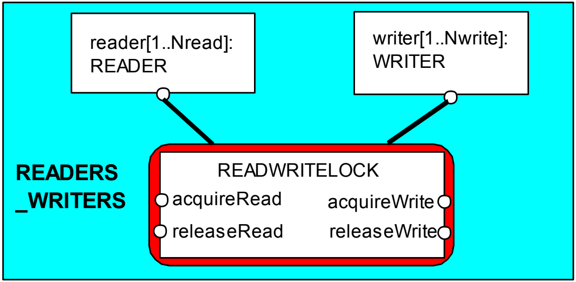
Readers and Writers

- A shared database is accessed by two kinds of processes. **Readers** execute transactions that examine the database while **Writers** both examine and update the database. A **Writer** must have exclusive access to the database; any number of **Readers** may concurrently access it.
- Events or actions of interest: *acquireRead*, *releaseRead*, *acquireWrite*, *releaseWrite*
- Processes: *Readers*, *Writers*, *RW_Lock*
- Properties: *RW_Safe*, *RW_Progress*



```
set Actions =
 {acquireRead,releaseRead,acquireWrite,releaseWrite}

READER = (acquireRead->examine->releaseRead->READER)
  + Actions
  \ {examine}.

WRITER = (acquireWrite->modify->releaseWrite->WRITER)
  + Actions
  \ {modify}.
```

Alphabet extension is used to ensure that the other access actions cannot occur freely for any prefixed instance of the process (as before).

Action hiding is used as actions **examine** and **modify** are not relevant for access synchronisation.

```
const False = 0    const True  = 1
range Bool  = False..True
const Nread = 2              // Maximum readers
const Nwrite= 2              // Maximum writers

RW_LOCK = RW[0][False],
RW[readers:0..Nread][writing:Bool] =
      (when (!writing)
           acquireRead  -> RW[readers+1][writing]
      |releaseRead      -> RW[readers-1][writing]
      |when (readers==0 && !writing)
           acquireWrite -> RW[readers][True]
      |releaseWrite     -> RW[readers][False]
      ).
```

The lock maintains a count of the number of readers, and a Boolean for the writers.

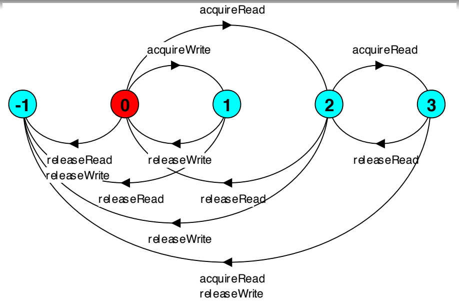
Safety Property

```
property SAFE_RW
   = (acquireRead  -> READING[1]
     |acquireWrite -> WRITING
     ),
READING[i:1..Nread]
   = (acquireRead -> READING[i+1]
     |when(i>1) releaseRead  -> READING[i-1]
     |when(i==1) releaseRead -> SAFE_RW
     ),
WRITING = (releaseWrite -> SAFE_RW).
```

We can check that **RW_LOCK** satisfies the safety property......

```
||READWRITELOCK = (RW_LOCK || SAFE_RW).
```

- Note that we do not check this property for the whole system, only for one component namely *RW_LOCK*. This is computationally simpler.



- An *ERROR* occurs if a reader or writer is badly behaved (*release* before *acquire* or more than two readers).
- However when composing with *READWRITELOCK* such bad behaviour is not allowed.

```
||READERS_WRITERS
   = (reader[1..Nread] :READER
     || writer[1..Nwrite]:WRITER
     ||{reader[1..Nread],
        writer[1..Nwrite]}::READWRITELOCK).
```

Safety and Progress Analysis ?

```
||READERS_WRITERS
   = (reader[1..Nread] :READER
     || writer[1..Nwrite]:WRITER
     ||{reader[1..Nread],
        writer[1..Nwrite]}::READWRITELOCK).
```

Safety and Progress Analysis ?

- Neither deadlock nor safety violation.
- It requires a tool to show it, the tool is not efficient (it **cannot** be).
- Try the tool for 10 readers and 10 writers!
- It is always better if some properties can just be **proved**, **not only checked**.
- **Problem with checking: one cannot checked the case of** n **readers and** m **writers, only, say, 5 readers and 4 writers, etc.**

Liveness

```
progress WRITE = {writer[1..Nwrite].acquireWrite}
progress READ  = {reader[1..Nread].acquireRead}
```

WRITE - eventually one of the writers will *acquireWrite*

READ - eventually one of the readers will *acquireRead*

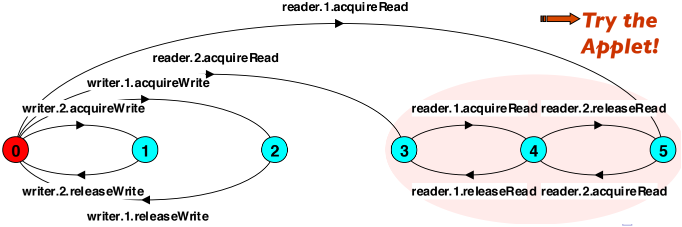
- I do not like it! Why not all?
- Actually this problem shows well the limits of pure FSP model.

- No *FAIR CHOICE* assumption: both *write* and *read* can starve.
- *FAIR CHOICE* assumption: both *write* and *read* are live.
- But waht in *real world* th assumption of *FAIR CHOICE* mean?
- This is assumption about a *possibility* of *clever implementation*.

- Simple use of priorities does not guarantee liveness.
- We lower the priority of the release actions for both *readers* and *writers*.

```
||RW_PROGRESS = READERS_WRITERS
                >>{reader[1..Nread].releaseRead,
                   writer[1..Nwrite].releaseWrite}.
```

- Progress violation: *WRITE*
- Path to terminal set of states: *reader.1.acquireRead*
- Actions in terminal set: {*reader.1.acquireRead*, *reader.1.releaseRead*, *reader.2.acquireRead*, *reader.2.releaseRead*}
- *WRITER* starvation: the number of readers never drops to zero!



Try the Applet!

WRITER Priority

- Block readers if there is a writer waiting.

```
RW_LOCK = RW[0][False][0],
RW[readers:0..Nread][writing:Bool][waitingW:0..Nwrite]
= (when (!writing && waitingW==0)
      acquireRead -> RW[readers+1][writing][waitingW]
   |releaseRead -> RW[readers-1][writing][waitingW]
   |when (readers==0 && !writing)
      acquireWrite-> RW[readers][True][waitingW-1]
   |releaseWrite-> RW[readers][False][waitingW]
   |requestWrite-> RW[readers][writing][waitingW+1]
   ).
```

property `RW_SAFE`:

```
No deadlocks/errors
```

progress `READ` **and** `WRITE`:

```
Progress violation: READ
Path to terminal set of states:
     writer.1.requestWrite
     writer.2.requestWrite
Actions in terminal set:
{writer.1.requestWrite, writer.1.acquireWrite,
 writer.1.releaseWrite, writer.2.requestWrite,
 writer.2.acquireWrite, writer.2.releaseWrite}
```

Reader starvation: ***if*** always a writer waiting.

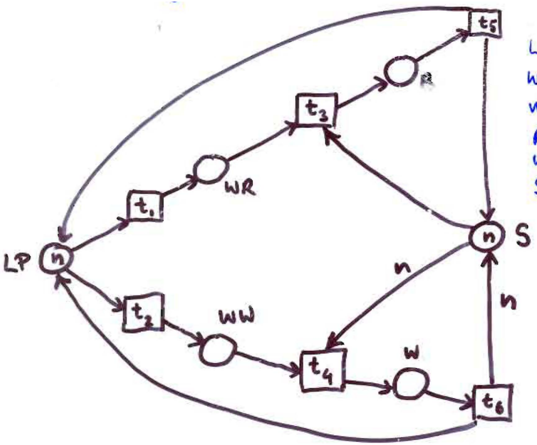
In practice, this may be satisfactory as is usually more read access than write, and readers generally want the most up to date information.

- We have encountered many problems both with formulation of the problem and solutions to it.
- Let us try another formalism.

Readers and Writers Again

- We have n processes, $n > 0$, which may read and write in a shared memory. Several precesses may be reading concurrently, but when a process is writing, no other process can be reading or writing. No priority is assumed.

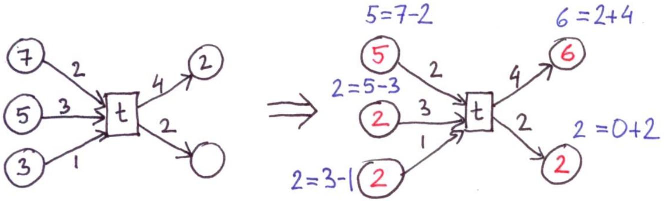
Readers and Writers: P/T Net Model



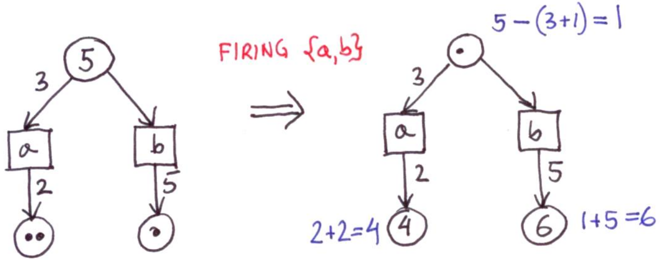
LP – local processing
WR – waiting to read
WW – waiting to write
R – reading
W – writing
S – synchronisation

Place/Transitions Nets (P/T-Nets)

- Firing rules:



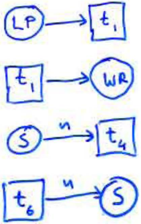
- Different kind of simultaneity:



Incidence Matrix

- P - places, T - transitions

								INVARIANTS		
P \ T	t_1	t_2	t_3	t_4	t_5	t_6	m_0	i_1	i_2	i_3
LP	-1	-1		1	1		n	1		-1
WR	1		-1					1		-1
WW		1		-1				1		-1
R			1		-1			1	1	
W				1		-1		1	n	n-1
S			-1	-n	1	n	n		1	1



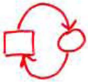
$\Longleftrightarrow$

$w(t_1, LP) = -1$

$w(t_1, WR) = 1$

$w(t_4, S) = -n$

$w(t_6, S) = n$



disallowed!

Multisets (Bags, Weighted Sets)

- A multiset m, over a non-empty and finite set S is a function $m : S \to \mathbb{N} = \{0, 1, 2, \dots\}$
- $m(s)$ is the number of appearances of s in m.
- notation: M is usually represented by:

$$\sum_{s \in S} m(s)s$$

$S = \{a, b, c, d, e\}$,
$m(a) = 3, m(b) = 1, m(c) = 0, m(d) = 183, m(e) = 4$

$$m = 3a + b + 4e + 183d$$

- $s \in m \iff m(s) \neq 0$
- $m(s)$ is a *coefficient*
- the *empty multiset* $m = \emptyset \iff m(s)$ for each $s \in S$.

Basic Definitions

- Let $\mathbf{x}$ be a **multiset (weighted set) of transitions**, i.e.
  $\mathbf{x} : T \to \mathbb{N}$
- $\mathbf{x}$ is **positive** iff $\mathbf{x}(t) > 0$ for at least one $t \in T$, i.e. $\mathbf{x} \neq \emptyset$
- **Marking**: $m : P \to \mathbb{N}$.
  Marking **is not** interpreted as a multiset!
- $m \geq m' \iff \forall p \in P.\ m(p) \geq m'(p)$.
- **Assumption:** Each place can hold an arbitrary number of tokens.
- Let W^- be the following matrix:

$$\forall (p,t) \in P \times T.\ W^-(p,t) = \begin{cases} -W(p,t) & \text{if } W(p,t) < 0 \\ 0 & \text{if } W(p,t) \geq 0 \end{cases}$$

- A positive multiset of transitions $\mathbf{x}$ has **concession** in a marking m iff $m \geq W^- \cdot \mathbf{x}$

$\uparrow$

matrix multiplication

Example ($n = 15$)

- $\mathbf{x} = 10t_1 + 3t_2$ **has** a concession in $m_0 = (15, 0, 0, 0, 0, 15)$, since

$$W^- \cdot \mathbf{x} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 15 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = (13, 0, 0, 0, 0, 0),$$

  and $m_0 > (13, 0, 0, 0, 0, 0)$.

- $\mathbf{x} = t_4$ **does not have** a concession in $m = (8, 3, 1, 2, 0, 13)$, since

$$W^- \cdot \mathbf{x} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 15 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = (0, 0, 1, 0, 0, 15),$$

  and m and $(0, 0, 1, 0, 0, 15)$ are incomparable.

Firing

- When $\mathbf{x}$ has concession, it may **fire**.
- If $\mathbf{x}$ fires, w new marking:

$$m' = m + W \cdot \mathbf{x}$$

  is reached.
- m' is said to be **directly reachable** from m , i.e. $m \Rightarrow m'$
- $\Rightarrow^* = \bigcup_{i=0}^{\infty} \Rightarrow^i$, or $\Rightarrow^*$ is a reflexive and transitive closure of $\Rightarrow$, is called **reachability**.

Example

Let $m_0 = (15, 0, 0, 0, 0, 15)$ and $\mathbf{x} = 10t_1 + 3t_2$.
We calculate $m_1 = m_0 + W \cdot \mathbf{x}$.

$$m_1 = m_0 + W \cdot \mathbf{x} =$$

$$(15, 0, 0, 0, 0, 15) + \begin{bmatrix} -1 & -1 & 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -15 & 1 & 15 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} =$$

$$(15, 0, 0, 0, 0, 15) + (-13, 10, 3, 0, 0, 0) = (2, 10, 3, 0, 0, 15).$$

Invariants

- Let $\mathbf{v}$ be a **multiset of places**, i.e. $\mathbf{v} : P \to \mathbb{N}$.

Note that $m : P \to \mathbb{N}$ and $\mathbf{v} : P \to \mathbb{N}$,but the interpretation is different, marking is not interpreted as a multiset!

Theorem (Lautenbach 1979)

Let $\mathbf{v}$ be a multiset of places. If $\mathbf{v} \cdot W = 0$ and $m \Rightarrow^* m'$ then

$$v \cdot m' = v \cdot m.$$

Proof.

It suffices to show it for $m \Rightarrow m'$. $v \cdot m' = v \cdot (m + W \cdot \mathbf{x}) = v \cdot m + v \cdot (W \cdot \mathbf{x}) = v \cdot m + (v \cdot W) \cdot \mathbf{x} = v \cdot m + 0 \cdot \mathbf{x} = v \cdot m$. □

Definition

A multiset of places $\mathbf{v}$ is said to be an **invariant** iff $v \cdot W = 0$.

- Each linear combination of invariants is itself an invariant.

Multiplication of a Vector by an Array

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = (a_1 b_{11} + a_1 b_{12} + a_1 b_{13},\ a_2 b_{21} + a_2 b_{22} + a_2 b_{23},\ a_3 b_{31} + a_3 b_{32} + a_3 b_{33})$$

Example

Consider $i_1 = (1,1,1,1,1,0)$, $i_2 = (0,0,0,1,n,1)$, $i_3 = (-1,-1,-1,0,n-1,1)$. We show that i_1, i_2 and i_3 are invariants.

$$i_1 \cdot W = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & -1 & 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -n & 1 & n \end{bmatrix} = (0,0,0,0,0,0)$$

$$i_2 \cdot W = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ n \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & -1 & 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -n & 1 & n \end{bmatrix} = (0,0,0,0,0,0)$$

$i_3 = i_2 - i_1$.

Invariant As an Expression

Definition

An invariant can also be defined as a **formula** obtained from $\mathbf{v} \cdot m_0 = \mathbf{v} \cdot m$, where $\mathbf{v}$ is an invariant, as defined previously, m_0 is the initial marking, and m is a marking variable.

Example

$i_1 = (1,1,1,1,1,0),\ m_0 = (n,0,0,0,0,n)$.
$i_1 \cdot m_0 = (1,1,1,1,1,0) \cdot (n,0,0,0,0,n) = n$
$m = (m(LP), m(WR), m(WW), m(R), m(W), m(S))$
$i_1 \cdot m =$
$(1,1,1,1,1,0) \cdot (m(LP), m(WR), m(WW), m(R), m(W), m(S)) =$
$m(LP) + m(WR) + m(WW) + m(R) + m(W)$.
$i_1 \cdot m_0 = i_1 \cdot m \implies$
$m(LP) + m(WR) + m(WW) + m(R) + m(W) = n$

- **The number of processes is an invariant.**

Example

$i_2 = (0,0,0,1,n,1),\ m_0 = (n,0,0,0,0,n)$.
$m = (m(LP), m(WR), m(WW), m(R), m(W), m(S))$
$i_2 \cdot m_0 = (0,0,0,1,n,1) \cdot (n,0,0,0,0,n) = n$
$i_2 \cdot m =$
$(0,0,0,1,n,1) \cdot (m(LP), m(WR), m(WW), m(R), m(W), m(S)) =$
$m(R) + n \cdot m(W) + m(S)$.

$$i_2 \cdot m_0 = i_2 \cdot m \implies m(R) + n \cdot m(W) + m(S) = n$$

- **When a process is writing, no other process can be reading or writing.**
- **The number of reading processes is between 0 and n.**
- **If no process is reading and writing, $m(S) = n$.**
- t_3 has concession if at least one process is waiting to read.
- t_4 has concession if at least one process is waiting to write.

Example

$i_3 = (-1,-1,-1,0,n-1,1),\ m_0 = (n,0,0,0,0,n)$.
$m = (m(LP), m(WR), m(WW), m(R), m(W), m(S))$
$i_3 \cdot m_0 = (-1,-1,-1,0,n-1,1) \cdot (n,0,0,0,0,n) = 0$
$i_3 \cdot m = (-1,-1,-1,0,n-1,1) \cdot$
$(m(LP), m(WR), m(WW), m(R), m(W), m(S)) =$
$-m(LP) - m(WR) - m(WW) + (n-1)m(W) + m(S)$

$$i_3 \cdot m_0 = i_3 \cdot m \implies m(LP) + m(WR) + m(WW) = (n-1)m(W) + m(S)$$

- **If no process is writing then $m(WR) \leq m(S)$.**
- t_3 has concession if at least one process is waiting to read.

Deadlock-freeness of Readers and Writers

Proposition

The Readers-Writers net cannot deadlock (reach a marking where no transition has concession).

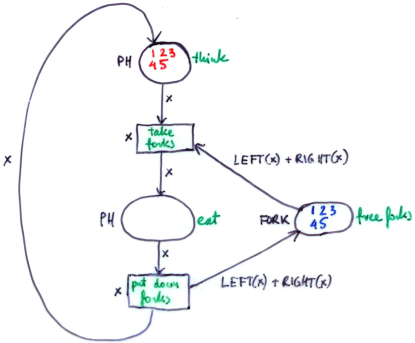
Proof.

If $m(LP) + m(R) + m(W) > 0$, it follows form the fact that t_1, t_2, t_5 or t_6 has concession.
If $m(LP) + m(R) + m(W) = 0$, it follows from i_1 and i_2 as they imply:

$$m(WR) + m(WW) = n$$
$$m(S) = n$$

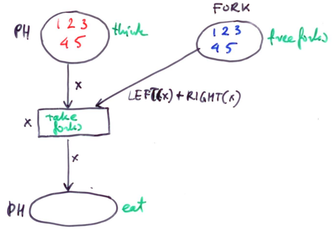
so t_3 or t_4 have concession. □

Invariants for Coloured Petri Nets: Dining Philosophers



$colour\ PH = with\ ph1 \mid ph2 \mid ph3 \mid ph4 \mid ph5$
$colour\ Fork = with\ f1 \mid f2 \mid f3 \mid f4 \mid f5$
$LEFT : PH \rightarrow FORK, \quad RIGHT : PH \rightarrow FORK$
$var\ x : \ PH$
$fun\ LEFT\ x = case\ of\ ph1 \Rightarrow f2 \mid ph2 \Rightarrow f3 \mid ph3 \Rightarrow f4 \mid$
$ph4 \Rightarrow f5 \mid ph5 \Rightarrow f1$
$fun\ RIGHT\ x = case\ of\ ph1 \Rightarrow f1 \mid ph2 \Rightarrow f2 \mid ph3 \Rightarrow f3 \mid$
$ph4 \Rightarrow f4 \mid ph5 \Rightarrow f5$

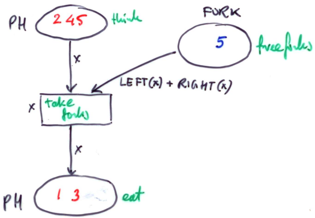
Firing



$\Downarrow$

Firing occurrence: $(\mathit{take\ forks}, \underbrace{x = ph1}_{\textit{binding}}) + (\mathit{take\ forks}, \underbrace{x = ph3}_{\textit{binding}})$

$\Downarrow$



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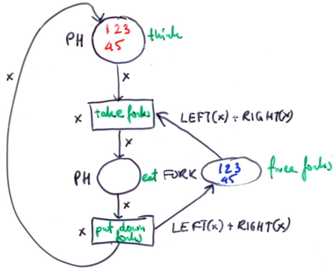
$$m = 3a + b + 4e + 183d$$

- $s \in m \iff m(s) \neq 0$
- $m(s)$ is a *coefficient*
- the *empty multiset* $m = \emptyset \iff m(s)$ for each $s \in S$.

Behaviours



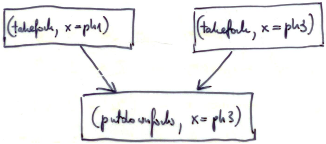
Sequence:

$(take\ forks, x = ph1)(take\ forks, x = ph3)(putdown\ forks, x = ph3)$

Step-sequence:

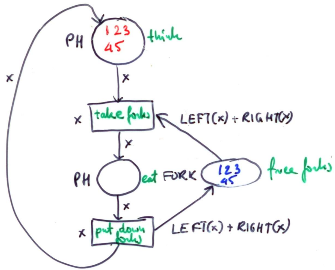
$\{(take\ forks, x = ph1)(take\ forks, x = ph3)\}\{(putdown\ forks, x = ph3)\}$

Partial order:

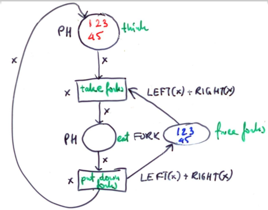


Invariants

- Invariants are equations that characterize all reachable markings.



- $M(think) + M(eat) = ph1 + ph2 + ph3 + ph4 + ph5$
  Each philosopher is either thinking or eating but not both. Also philosophers do not disappear and no new is born.
- $LEFT(M(eat)) + RIGHT(M(eat)) + M(free\ forks) = f_1 + f_2 + f_3 + f_4 + f_5$
  where $LEFT(X) = \sum_{x \in X} LEFT(x)$, $RIGHT(X) = \sum_{x \in X} RIGHT(x)$
  No philosopher can be eating at the same time as on of his neighbours.



(i1) $M(think) + M(eat) = PH$

(i2) $LEFT(M(eat)) + RIGHT(M(eat)) + M(free\ forks) = FORK$

Proposition

The above Coloured Petri net cannot deadlock.

Proof.

Assume that M is reachable from the initial marking. Then M satisfies (i1) and (i2).
If $M(eat) \neq \emptyset$, i.e. $phj \in M(eat)$, then $(putdown\ fork, x = phj)$ can be fired.
If $M(eat) = \emptyset$ it follows from (i1) and (i2) that

$$M(think) = PH \text{ and } M(free\ forks) = FORK$$

Then $(take\ forks, x = phi)$, any $phi \in PM$ can be fired. □

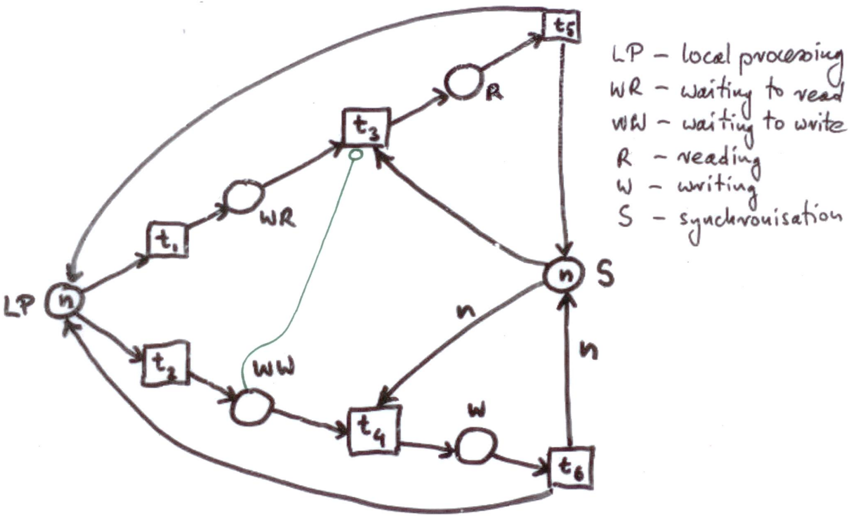
How to Find Invariants?

- Finding invariants can be reduced to finding non-negative integer solutions of some matrix equation:

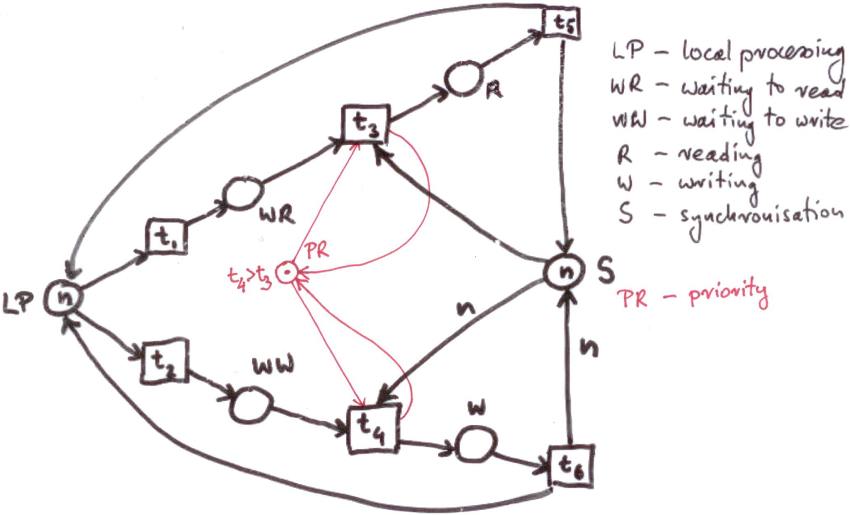
$$W \cdot X = \mathbf{0}$$

  where $\mathbf{0}$ is a vector of zeros,
  W represents the structure of a net (incidence matrix),
  X represents an invariant.
- The number of invariants is infinite, but there is a finite number of linearly independent invariants
- Proper invariants are part of specification goals.
- Checking if an equation is an invariant is easy!

Writers Priority with Inhibitor Arcs



Writers Priority with Selfloops



Writers Priority without Selfloops