

Deadlock

CS 2SD3

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Concepts: system **deadlock**: no further progress
four necessary & sufficient conditions

Models: deadlock - no eligible actions

Practice: blocked threads

Aim: deadlock avoidance - to design systems where deadlock cannot occur.

Deadlock: four necessary and sufficient conditions

◆ Serially reusable resources:

*the processes involved share resources which they use under **mutual exclusion**.*

◆ Incremental acquisition:

processes hold on to resources already allocated to them while waiting to acquire additional resources.

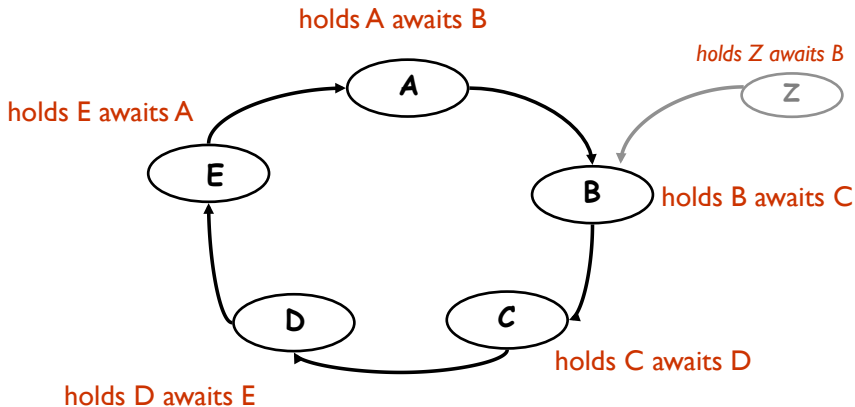
◆ No pre-emption:

once acquired by a process, resources cannot be pre-empted (forcibly withdrawn) but are only released voluntarily.

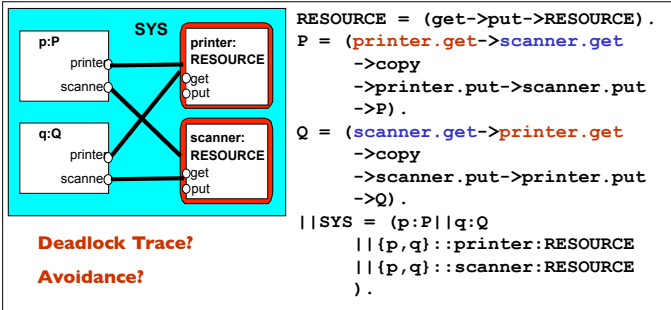
◆ Wait-for cycle:

a circular chain (or cycle) of processes exists such that each process holds a resource which its successor in the cycle is waiting to acquire.

Wait-for cycle



Deadlock may arise from the parallel composition of interacting processes.



$p : P = (p.\text{printer.get} \bullet \rightarrow p.\text{scanner.get} \rightarrow p.\text{copy} \rightarrow p.\text{printer.put} \rightarrow p.\text{scanner.put} \rightarrow p : P)$

$q : Q = (q.\text{scanner.get} \bullet \rightarrow q.\text{printer.get} \rightarrow q.\text{copy} \rightarrow q.\text{scanner.put} \rightarrow q.\text{printer.put} \rightarrow q : Q)$

$\underbrace{\{p, q\} :: \text{printer} : \text{RESOURCE}}_{pqpR} = (p.\text{printer.get} \bullet \rightarrow p.\text{printer.put} \rightarrow pqpR \mid q.\text{printer.get} \rightarrow q.\text{printer.put} \rightarrow pqpR)$

$\underbrace{\{p, q\} :: \text{scanner} : \text{RESOURCE}}_{pqsR} = (p.\text{scanner.get} \rightarrow p.\text{scanner.put} \rightarrow pqsR \mid q.\text{scanner.get} \bullet \rightarrow q.\text{scanner.put} \rightarrow pqsP)$

Deadlock sequence: $p.\text{printer.get} \rightarrow q.\text{scanner.get}$

• - denote states where processes deadlock

A Possible Solutions

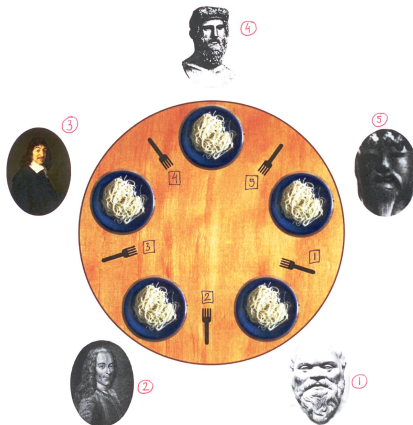
- Acquire resources in the same order, i.e. printers always before scanners.
- Timeout:

```
P          = (printer.get-> GETSCANNER) ,  
GETSCANNER = (scanner.get->copy->printer.put  
              ->scanner.put->P  
              | timeout -> printer.put->P  
              ) .  
  
Q          = (scanner.get-> GETPRINTER) ,  
GETPRINTER = (printer.get->copy->printer.put  
              ->scanner.put->Q  
              | timeout -> scanner.put->Q  
              ) .
```

- No deadlock but the sequence:
printer.get → *timeout* → *printer.put* →
can be repeated infinite number of times!
NOBODY COPIES ANYTHING!

Dining Philosophers

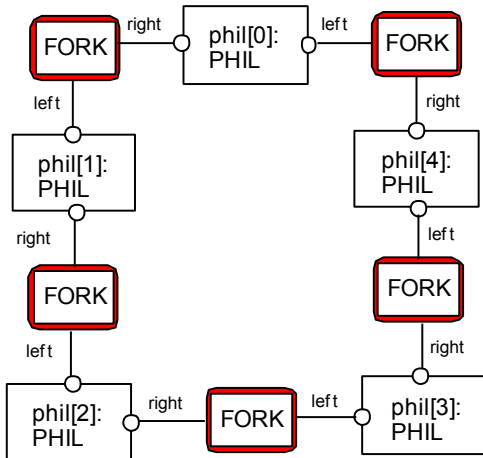
- Five philosophers sit around a circular table. Each philosopher spends his life alternately **thinking** and **eating**. To eat, a philosopher needs **two forks**, but unfortunately there are only five forks on the circular table and each philosopher is only allowed to use the two forks nearest to him.



Dining Philosophers - model structure diagram

Each FORK is a **shared resource** with actions **get** and **put**.

When hungry, each PHIL must first get his right and left forks before he can start eating.



Hungry, Simple Minded Philosophers

- $i \oplus 1 = \text{if } i < 5 \text{ then } i + 1 \text{ else } 1$

$FORK = (\text{get} \rightarrow \text{put} \rightarrow FORK)$

$PHIL = (\text{think} \rightarrow \text{right.get} \rightarrow \text{left.get} \rightarrow \text{eat} \rightarrow \text{right.put} \rightarrow \text{left.put} \rightarrow PHIL)$

$\parallel DINERS(N = 5) = \text{forall}[i : 1..N]$
 $(\text{phil}[i] : PHIL \parallel \{\text{phil}[i].\text{right}, \text{phil}[i \oplus 1].\text{left}\} :: FORK)$

- More intuitively (for $\text{get}_j^i, \text{put}_j^i$, i - philosopher number, j - fork number):

$FORK_1 = (\text{get}_1^1 \rightarrow \text{put}_1^1 \rightarrow FORK_1 \mid \text{get}_1^5 \rightarrow \text{put}_1^5 \rightarrow FORK_1)$

$FORK_2 = (\text{get}_2^2 \rightarrow \text{put}_2^2 \rightarrow FORK_2 \mid \text{get}_2^1 \rightarrow \text{put}_2^2 \rightarrow FORK_2)$

$FORK_3 = (\text{get}_3^3 \rightarrow \text{put}_3^3 \rightarrow FORK_3 \mid \text{get}_3^2 \rightarrow \text{put}_3^3 \rightarrow FORK_3)$

$FORK_4 = (\text{get}_4^4 \rightarrow \text{put}_4^4 \rightarrow FORK_4 \mid \text{get}_4^3 \rightarrow \text{put}_4^4 \rightarrow FORK_4)$

$FORK_5 = (\text{get}_5^5 \rightarrow \text{put}_5^5 \rightarrow FORK_5 \mid \text{get}_5^4 \rightarrow \text{put}_5^5 \rightarrow FORK_5)$

$PHIL_1 = (\text{think}_1 \rightarrow \text{get}_1^1 \rightarrow \text{get}_2^1 \rightarrow \text{eat}_1 \rightarrow \text{put}_1^1 \rightarrow \text{put}_2^1 \rightarrow PHIL_1)$

$PHIL_2 = (\text{think}_2 \rightarrow \text{get}_2^2 \rightarrow \text{get}_3^2 \rightarrow \text{eat}_2 \rightarrow \text{put}_2^2 \rightarrow \text{put}_3^2 \rightarrow PHIL_2)$

$PHIL_3 = (\text{think}_3 \rightarrow \text{get}_3^3 \rightarrow \text{get}_4^3 \rightarrow \text{eat}_3 \rightarrow \text{put}_3^3 \rightarrow \text{put}_4^3 \rightarrow PHIL_3)$

$PHIL_4 = (\text{think}_4 \rightarrow \text{get}_4^4 \rightarrow \text{get}_5^4 \rightarrow \text{eat}_4 \rightarrow \text{put}_4^4 \rightarrow \text{put}_5^4 \rightarrow PHIL_4)$

$PHIL_5 = (\text{think}_5 \rightarrow \text{get}_5^5 \rightarrow \text{get}_1^5 \rightarrow \text{eat}_5 \rightarrow \text{put}_5^5 \rightarrow \text{put}_1^5 \rightarrow PHIL_5)$

$\parallel DINERS = (FORK_1 \parallel \dots \parallel FORK_5 \parallel PHIL_1 \parallel \dots \parallel PHIL_5)$

- **Obvious deadlock! Everyone picks right fork.**

Trace₁ =

phil.1.think → phil.1.right.get →
phil.2.think → phil.2.right.get →
phil.3.think → phil.3.right.get →
phil.4.think → phil.4.right.get →
phil.5.think → phil.5.right.get

think₁ → get₁¹ →
think₂ → get₂¹ →
think₃ → get₃¹ →
think₄ → get₄¹ →
think₅ → get₅¹

What if not 'Simple Minded'?

$FORK = (get \rightarrow put \rightarrow FORK)$

$PHIL = THINK$

$THINK = (think \rightarrow$
 $(right.get \rightarrow left.get \rightarrow EAT \mid left.get \rightarrow right.get \rightarrow EAT))$

$EAT = (eat \rightarrow$
 $(right.put \rightarrow left.pt \rightarrow THINK \mid left.put \rightarrow right.put \rightarrow THINK$

$\parallel DINERS(N = 5) = forall[i : 1..N]$
 $(phil[i] : PHIL \parallel \{phil[i].right, phil[i \oplus 1].left\} :: FORK)$

- Unfortunately a freedom of choosing either right or left fork does not solve the problem. The same trace leads to a deadlock. However in “real” implementation, it will make it happen less often.

Still 'Simple Minded' but not so 'Hungry'

$FORK = (get \rightarrow put \rightarrow FORK)$

$PHIL = THINK$

$THINK = (think \rightarrow right.get \rightarrow$
 $(left.get \rightarrow EAT \mid giveup \rightarrow right.put \rightarrow THINK))$

$EAT = (eat \rightarrow right.put \rightarrow left.put \rightarrow THINK)$

$\parallel DINERS(N = 5) = forall[i : 1..N]$
 $(phil[i] : PHIL \parallel \{phil[i].right, phil[i \oplus 1].left\} :: FORK)$

- **There is no deadlock now!**

$Trace_1 \rightarrow phil.i.giveup \rightarrow phil.i.right.put \rightarrow \dots$

- **However we might get:**

$Trace_1 \rightarrow Trace_2 \rightarrow$ and so on,

where: $Trace_2 = phil.1.giveup \rightarrow phil.1.right.put \rightarrow$
 $phil.2.giveup \rightarrow phil.2.right.put \rightarrow$
 $phil.3.giveup \rightarrow phil.3.right.put \rightarrow$
 $phil.4.giveup \rightarrow phil.4.right.put \rightarrow$
 $phil.5.giveup \rightarrow phil.5.right.put \rightarrow$

- **No philosopher will ever eat!**
Starvation!

'Hungry' and 'Asymmetrically Simple Minded', or 'Some Discipline Added'

- Philosophers 1, 3 and 5 always perform '*left.get* \rightarrow *right.get*', while 2 and 4 always perform '*right.get* \rightarrow *left.get*'.

FORK = (*get* \rightarrow *put* \rightarrow *FORK*)

PHIL = (*when*(*i* = 1 \vee *i* = 3 \vee *i* = 5) *think* \rightarrow *left.get* \rightarrow
right.get \rightarrow *eat* \rightarrow *left.put* \rightarrow *right.put* \rightarrow *PHIL*
| *when*(*i* = 2 \vee *i* = 4) *think* \rightarrow *right.get* \rightarrow
left.get \rightarrow *eat* \rightarrow *right.put* \rightarrow *left.put* \rightarrow *PHIL*)

\parallel *DINERS*(*N* = 5) = *forall*[*i* : 1..*N*]
(*phil*[*i*] : *PHIL* \parallel {*phil*[*i*].*right*, *phil*[*i* \oplus 1].*left*} :: *FORK*)

- Works! Neither deadlock nor starvation.**

Asymmetrically Simple Minded Philosophers

- Notation: for get_j^i, put_j^i , i - philosopher number, j - fork number

$FORK_1 = (get_1^1 \rightarrow put_1^1 \rightarrow FORK_1 \mid get_1^5 \rightarrow put_1^5 \rightarrow FORK_1)$

$FORK_2 = (get_2^2 \rightarrow put_2^2 \rightarrow FORK_2 \mid get_2^1 \rightarrow put_2^1 \rightarrow FORK_2)$

$FORK_3 = (get_3^3 \rightarrow put_3^3 \rightarrow FORK_3 \mid get_3^2 \rightarrow put_3^2 \rightarrow FORK_3)$

$FORK_4 = (get_4^4 \rightarrow put_4^4 \rightarrow FORK_4 \mid get_4^3 \rightarrow put_4^3 \rightarrow FORK_4)$

$FORK_5 = (get_5^5 \rightarrow put_5^5 \rightarrow FORK_5 \mid get_5^4 \rightarrow put_5^4 \rightarrow FORK_5)$

$PHIL_1 = (think_1 \rightarrow get_2^1 \rightarrow get_1^1 \rightarrow eat_1 \rightarrow put_2^1 \rightarrow put_1^1 \rightarrow PHIL_1)$

$PHIL_2 = (think_2 \rightarrow get_2^2 \rightarrow get_3^2 \rightarrow eat_2 \rightarrow put_2^2 \rightarrow put_3^2 \rightarrow PHIL_2)$

$PHIL_3 = (think_3 \rightarrow get_4^3 \rightarrow get_3^3 \rightarrow eat_3 \rightarrow put_4^3 \rightarrow put_3^3 \rightarrow PHIL_3)$

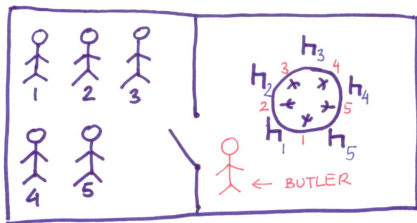
$PHIL_4 = (think_4 \rightarrow get_4^4 \rightarrow get_5^4 \rightarrow eat_4 \rightarrow put_4^4 \rightarrow put_5^4 \rightarrow PHIL_4)$

$PHIL_5 = (think_5 \rightarrow get_1^5 \rightarrow get_5^5 \rightarrow eat_5 \rightarrow put_1^5 \rightarrow put_5^5 \rightarrow PHIL_5)$

$\parallel DINERS = (FORK_1 \parallel \dots \parallel FORK_5 \parallel PHIL_1 \parallel \dots \parallel PHIL_5)$

'Hungry' and 'Simple Minded' but outside control, i.e. 'Butler'

- No more than 4 philosophers are sitting at the table.



$FORK = (get \rightarrow put \rightarrow FORK)$

$PHIL = (think \rightarrow sitdown \rightarrow right.get \rightarrow left.get \rightarrow eat \rightarrow right.put \rightarrow left.put \rightarrow getup \rightarrow PHIL)$

$BUTLER(K = 4) = COUNT[0]$

$COUNT[i : 1..4] = (when(i < K) sitdown \rightarrow COUNT[i + 1] | getup \rightarrow COUNT[i - 1])$

$\parallel DINERS(N = 5) = (forall[i : 1..N] (phil[i] : PHIL \parallel \{phil[i].right, phil[i \oplus 1].left\} :: FORK))$

$\parallel \underbrace{\{phil[i : ..N]\}}_{\text{philosophers}} :: BUTLER(K = 4))$

$\{phil[1], phil[2], phil[3], phil[4], phil[5]\}$

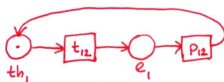
- 'Butler' solution works. No deadlock and no starvation.
- *FORK's* are *passive* processes (monitors), hence they always can be presented as:
$$FORK = (get \rightarrow put \rightarrow FORK)$$
- *PHILOSOPHER's* are *active* processes.

A Solution with Simultaneity

- No philosopher is allowed to grab one fork only, he must take both left and right at the same time if they are available.
- Modeling simultaneity is **not** *natural* in FSP approach, it is possible but looks artificial.
- Modeling simultaneity is *natural* when Petri nets are used.

Individual Philosophers and Free Forks as Petri Nets

- Philosopher No. 1:



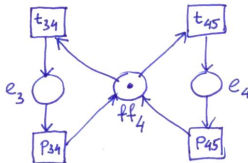
th_1 - Philosopher No.1 thinks

e_1 - Philosopher No.1 eats

t_{12} - Philosopher No.1 takes up forks 1 and 2

p_{12} - Philosopher No.1 puts down forks 1 and 2

- Free Fork No. 4:



ff_4 - Fork No. 4 is on the table

e_3 - Philosopher No. 3 eats using Fork No. 4

t_{34} - Fork No. 4 is taken by Philosopher No.3

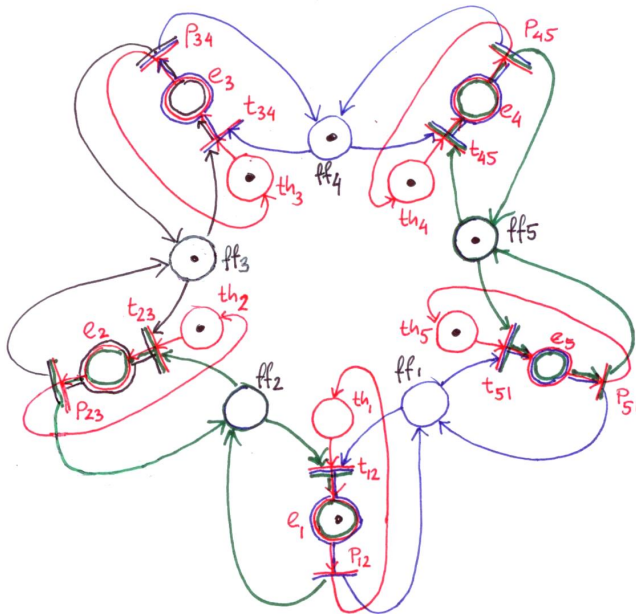
p_{34} - Fork No. 4 is put down Philosopher No.3

e_4 - Philosopher No. 4 eats using Fork No. 4

t_{45} - Fork No. 4 is taken by Philosopher No. 4

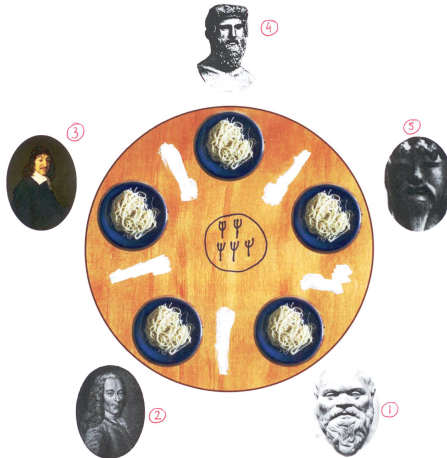
p_{45} - Fork No. 4 is put down Philosopher No. 4

Dining Philosophers Composed



A Solution with Simultaneity for FSP

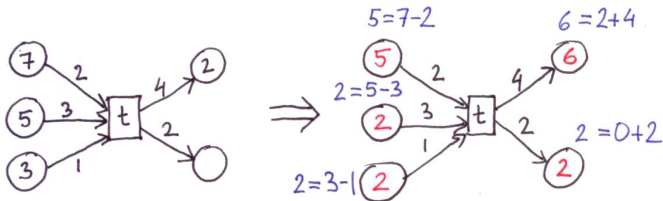
$$FORK = (take.right \rightarrow put.right \rightarrow FORK \mid \\ take.left \rightarrow put.left \rightarrow FORK)$$
$$PHIL = (think \rightarrow takeboth \rightarrow eat \rightarrow putboth \rightarrow PHIL)$$
$$\parallel DINERS(N = 5) = forall[i : 1..N] \\ (phil[i] : PHIL \parallel \{phil[i].right, phil[i \oplus 1].left\} :: FORK)) \\ / \{takeboth.1 / take.right.1, takeboth.1 / take.left.2, \\ takeboth.2 / take.right.2, takeboth.2 / take.left.3, \\ takeboth.3 / take.right.3, takeboth.3 / take.left.4, \\ takeboth.4 / take.right.4, takeboth.4 / take.left.5, \\ takeboth.5 / take.right.5, takeboth.5 / take.left.1, \\ putboth.1 / put.right.1, putboth.1 / put.left.2, \\ putboth.2 / put.right.2, putboth.1 / put.left.3, \\ putboth.3 / put.right.3, putboth.3 / put.left.4, \\ putboth.4 / put.right.4, putboth.4 / put.left.5, \\ putboth.5 / put.right.5, putboth.5 / put.left.1\}$$



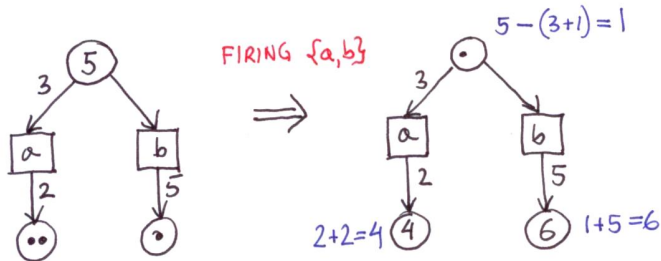
- All forks in one bowl. Forks are not distinguishable and philosophers pick them randomly.

Place/Transitions Nets (P/T-Nets)

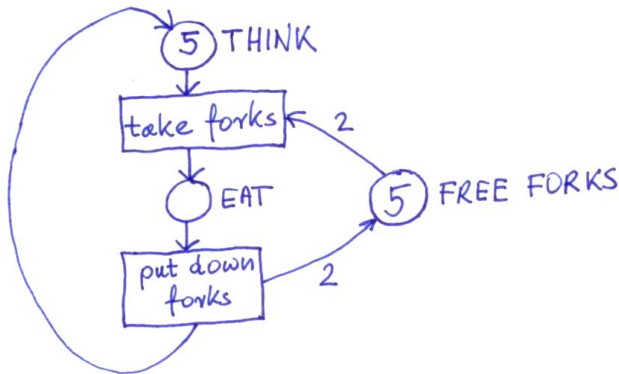
- Firing rules:



- Different kind of simultaneity:

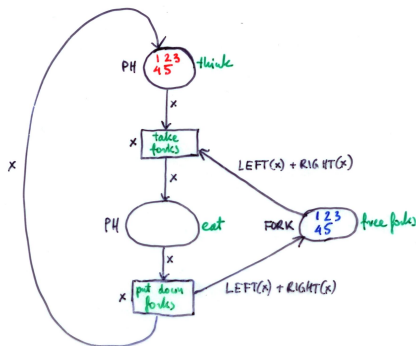


P/T-Nets Solution to 2nd Dining Philosophers Problem



- It does not work for the original Dining Philosophers Problem.
- Both philosophers and forks are represented by tokens.
- State machines represent *generic behaviours*.
- Impossible to model directly with FSP.

Coloured Petri Nets



colour $PH = with\ ph1 \mid ph2 \mid ph3 \mid ph4 \mid ph5$

colour $Fork = with\ f1 \mid f2 \mid f3 \mid f4 \mid f5$

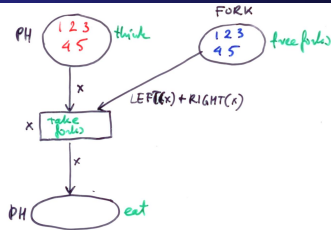
$LEFT : PH \rightarrow FORK, \quad RIGHT : PH \rightarrow FORK$

var $x : PH$

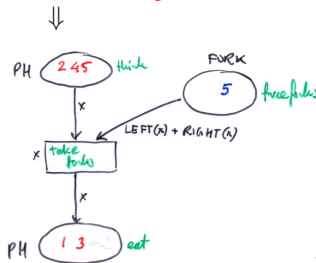
fun $LEFT\ x = case\ of\ ph1 \Rightarrow f2 \mid ph2 \Rightarrow f3 \mid ph3 \Rightarrow f4 \mid$
 $ph4 \Rightarrow f5 \mid ph5 \Rightarrow f1$

fun $RIGHT\ x = case\ of\ ph1 \Rightarrow f1 \mid ph2 \Rightarrow f2 \mid ph3 \Rightarrow f3 \mid$
 $ph4 \Rightarrow f4 \mid ph5 \Rightarrow f5$

Firing



Firing occurrence: $(take\ forks, \underbrace{x = ph1}_{binding}) + (take\ forks, \underbrace{x = ph3}_{binding})$



Multisets (or Bags)

- A multiset m , over a non-empty and finite set S is a function $m : S \rightarrow \mathbb{N} = \{0, 1, 2, \dots\}$
- $m(s)$ is the number of appearances of s in m .
- notation: M is usually represented by:

$$\sum_{s \in S} m(s)s$$

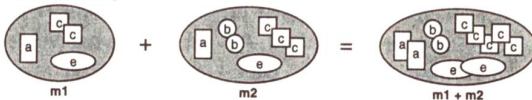
$$S = \{a, b, c, d, e\},$$
$$m(a) = 3, m(b) = 1, m(c) = 0, m(d) = 183, m(e) = 4$$

$$m = 3a + b + 183d + 4e$$

- $s \in m \iff m(s) \neq 0$
- $m(s)$ is a *coefficient*
- the *empty multiset* $m = \emptyset \iff m(s) = 0$ for each $s \in S$.

Some Operations on Multisets

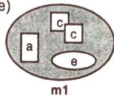
Addition (element-wise)



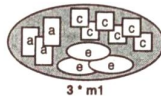
Scalar multiplication (element-wise)

3

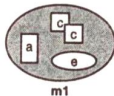
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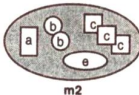
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Comparison (element-wise)



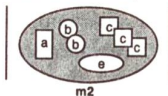
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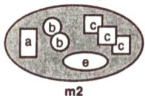
true

Size (number of elements)

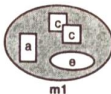


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Subtraction (only if $m2 \geq m1$)



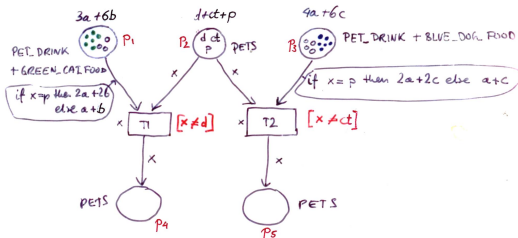
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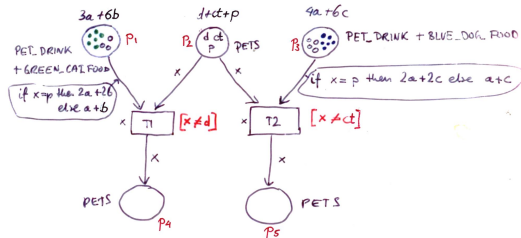


colour PET_DRINK = with a;
 colour GREEN_CAT_FOOD = with b;
 colour BLUE_DOG_FOOD = with c;
 colour PETS = with dog | cat | pig; (pig eats both cat food and dog food)
 var x : PETS; (in the drawing: dog=d, cat =ct, pig = p)



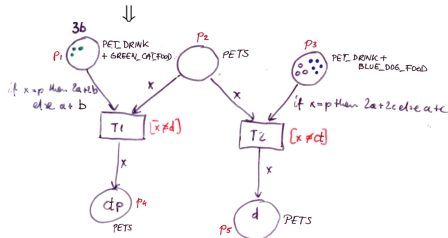
- In place p_1 we have 6 green cat food servings and 3 drinks (no dog food as it not allowed here, it is allowed in place p_3)
- Firing transition T_1 corresponds to allow cat or pig or both to eat. The cat eats 1 serving of cat food and 1 drink while the pig eats 2 servings of food and 2 drinks.
- If both cat and pig eat and drink, 3 drinks and 3 servings of cat food disappear from place p_1 , and p, ct disappear from place p_2 .
- Similarly for places p_3, p_5 and transition T_2 .

colour PET_DRINK = with a ;
 colour $GREEN_CAT_FOOD$ = with b ;
 colour $BLUE_DOG_FOOD$ = with c ;
 colour $PETS$ = with $dog \mid cat \mid pig$; (pig eats both cat food and dog food)
 var $x : PETS$; (in the drawing: $dog=d$, $cat=ct$, $pig=p$)

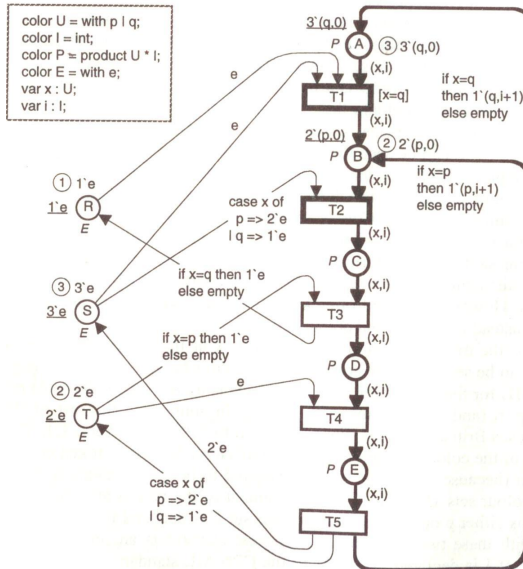


Firing occurrence: $(T_1, x = c) + (T_1, x = p) + (T_2, x = d)$

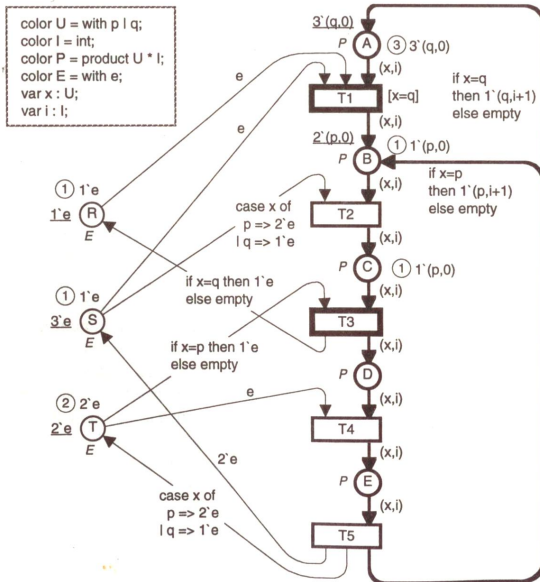
Interpretation: All three pets eat.



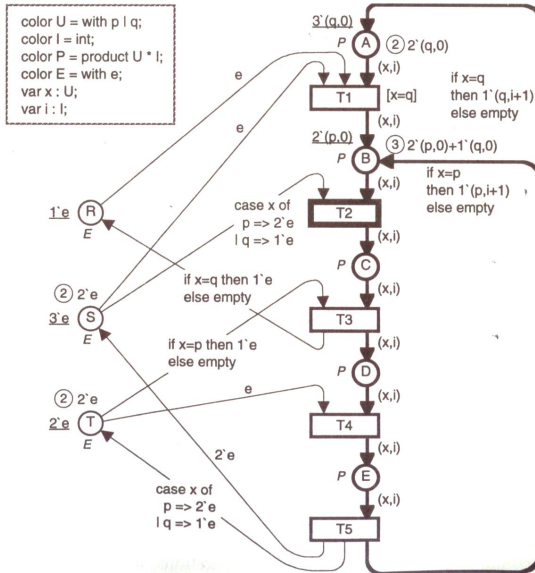
Some resource allocation system in its initial marking M_0



M_1 reachable from M_0 by $(T2, \langle x = p, i = 0 \rangle)$



M_2 reachable from M_0 by $(T2, \langle x = q, i = 0 \rangle)$



A Coloured Petri Net is a tuple: $N = (P, T, A, \Sigma, C, N, E, G, I)$ where:

- P is a set of places and T is a set of transitions.
- T is a set of transitions.
- A is a set of arcs
- In CPNs sets of places, transitions and arcs are pairwise disjoint
 $P \cap T = P \cap A = T \cap A = \emptyset$
- Σ is a set of color sets defined within CPN model. This set contains all possible colors, operations and functions used within CPN.
- C is a colour function. It maps places in P into colors in Σ .
- N is a node function. It maps A into $(P \times T) \cup (T \times P)$.
- E is an arc expression function. It maps each arc $a \in A$ into the expression e . The input and output types of the arc expressions must correspond to type of nodes the arc connected to.
- G is a guard function. It maps each transition $t \in T$ into guard expression g . The output of the guard expression should evaluate to Boolean value true or false.
- I is an initialization function. It maps each place p into an initialization expression i . The initialization expression must evaluate to multiset of tokens with a color corresponding to the

Some Concepts Needed

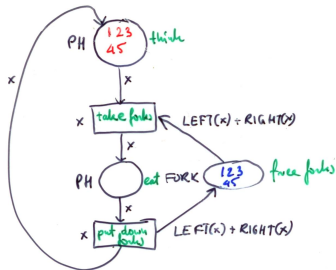
- The *elements of a type*, T . The set of all elements in T is denoted by the type name T itself.
- The *type of a variable*, v – denoted by $\text{Type}(v)$.
- The *type of an expression*, expr – denoted by $\text{Type}(\text{expr})$.
- The *set of variables in an expression*, expr – denoted by $\text{Var}(\text{expr})$.
- A *binding of a set of variables*, V – associating with each variable $v \in V$ an element $b(v) \in \text{Type}(v)$.
- The *value obtained by evaluating an expression*, expr , in a binding, b – denoted by $\text{expr}\langle b \rangle$. $\text{Var}(\text{expr})$ is required to be a subset of the variables of b , and the evaluation is performed by substituting for each variable $v \in \text{Var}(\text{expr})$ the value $b(v) \in \text{Type}(v)$ determined by the binding.

'Official' Formal Definition of Petri Nets

Definition 2.5: A **non-hierarchical CP-net** is a tuple $CPN = (\Sigma, P, T, A, N, C, G, E, I)$ satisfying the requirements below:

- (i) Σ is a finite set of non-empty types, called **colour sets**.
- (ii) P is a finite set of **places**.
- (iii) T is a finite set of **transitions**.
- (iv) A is a finite set of **arcs** such that:
 - $P \cap T = P \cap A = T \cap A = \emptyset$.
- (v) N is a **node** function. It is defined from A into $P \times T \cup T \times P$.
- (vi) C is a **colour** function. It is defined from P into Σ .
- (vii) G is a **guard** function. It is defined from T into expressions such that:
 - $\forall t \in T: [Type(G(t)) = \mathbb{B} \wedge Type(Var(G(t))) \subseteq \Sigma]$.
- (viii) E is an **arc expression** function. It is defined from A into expressions such that:
 - $\forall a \in A: [Type(E(a)) = C(p(a))_{MS} \wedge Type(Var(E(a))) \subseteq \Sigma]$
where $p(a)$ is the place of $N(a)$.
- (ix) I is an **initialization** function. It is defined from P into closed expressions such that:
 - $\forall p \in P: [Type(I(p)) = C(p)_{MS}]$.

Behaviours



Sequence:

$(take\ forks, x = ph1)(take\ forks, x = ph3)(putdown\ forks, x = ph3)$

Step-sequence:

$\{(take\ forks, x = ph1)(take\ forks, x = ph3)\}\{(putdown\ forks, x = ph3)\}$

Partial order:

