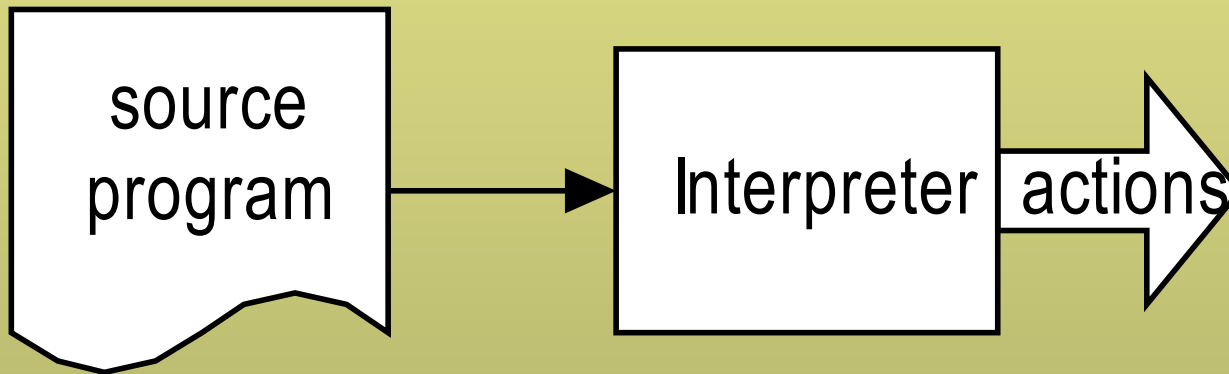
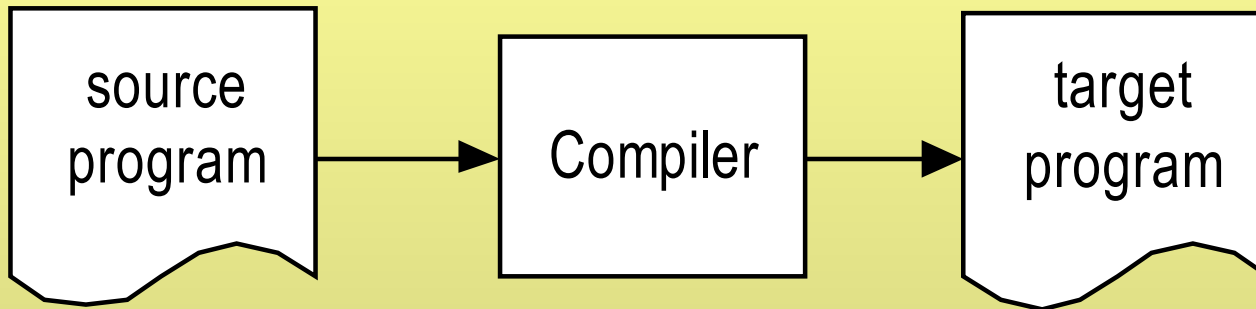


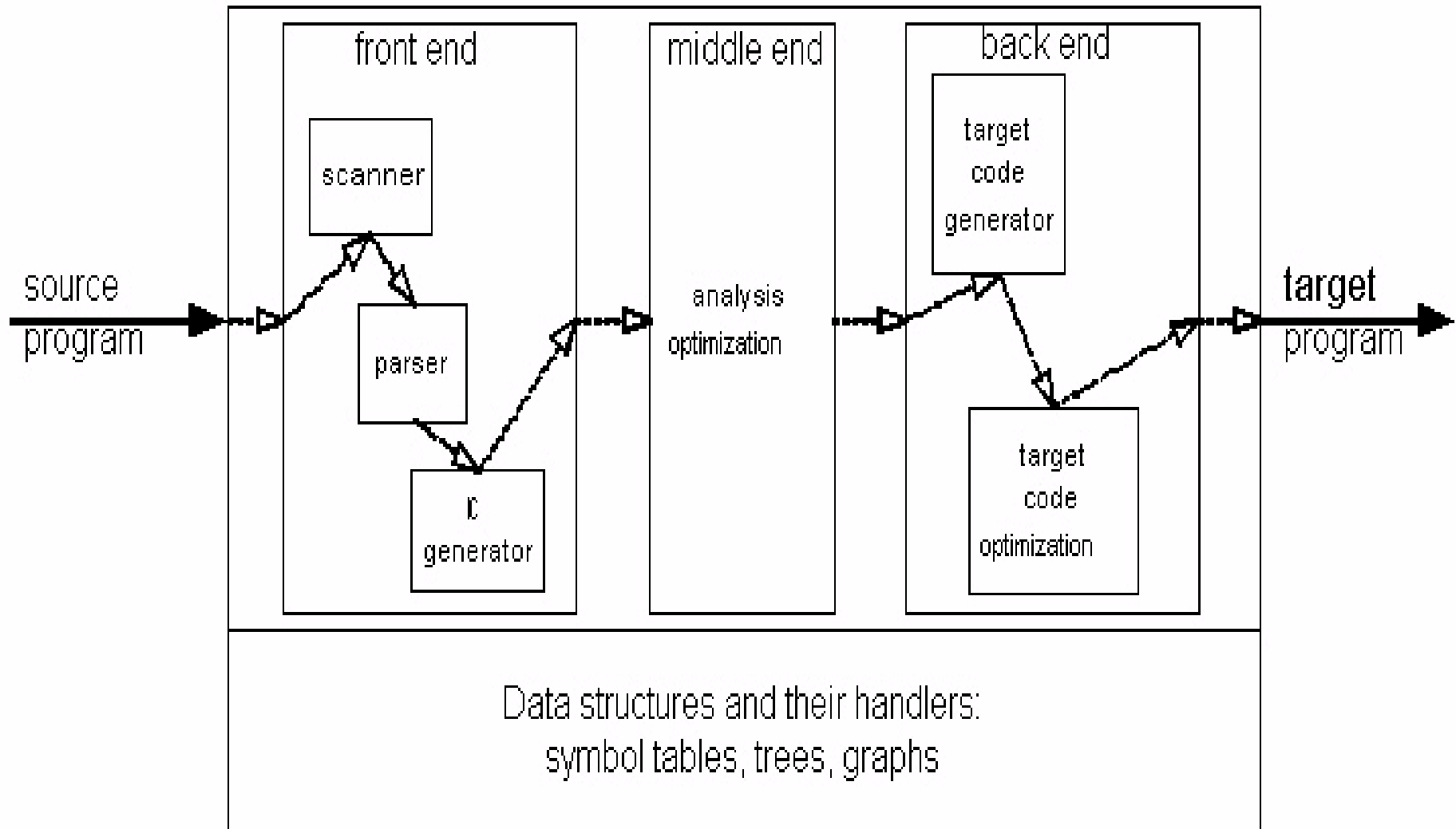
Compiler Optimization

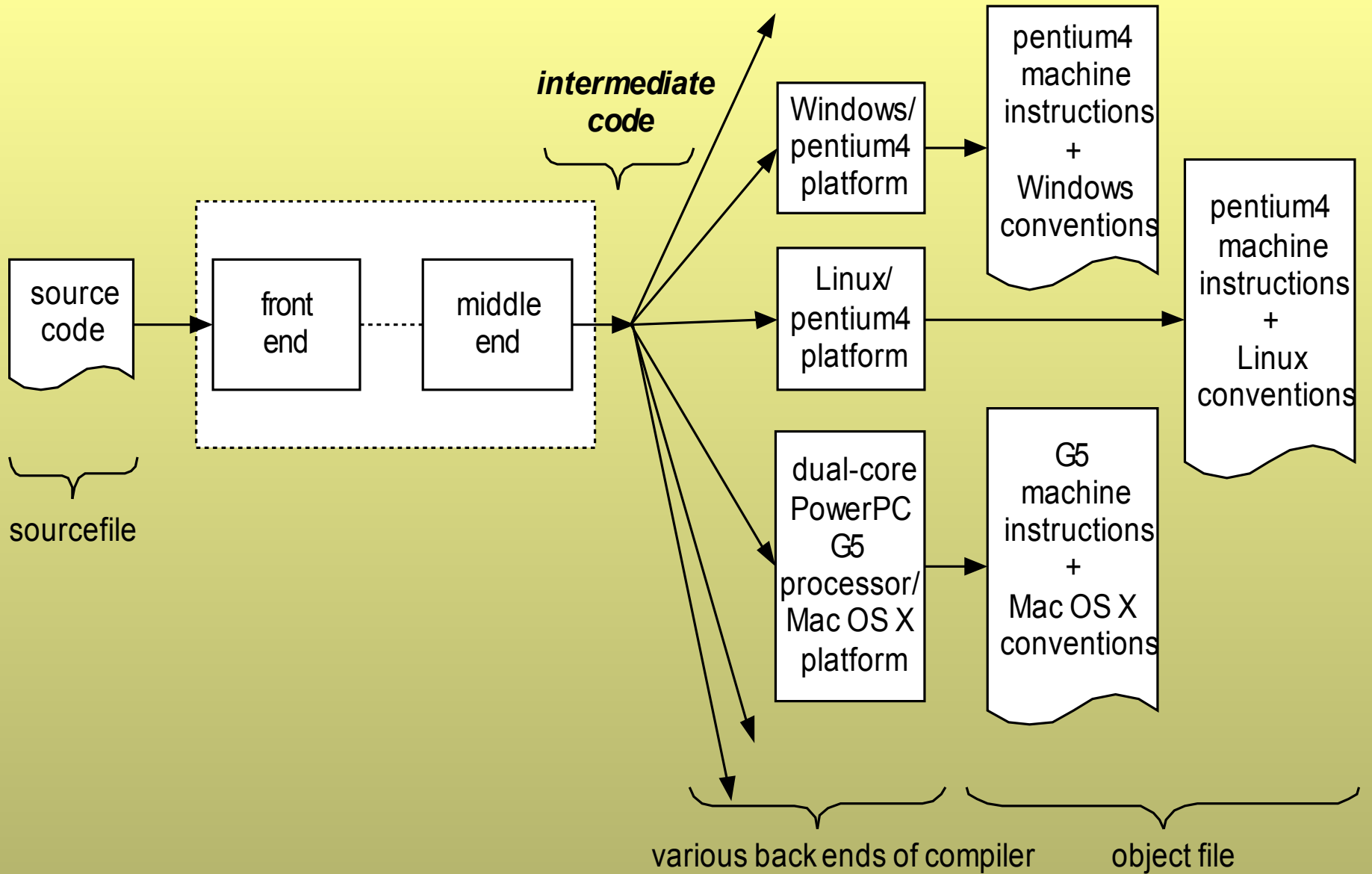
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- **Compilation phases**
- **Intermediate code**
- **Control flow analysis**
- **Data flow analysis**
- **Various optimization techniques:**
 - Peephole Optimization, Dead-code Elimination**
 - Unreachable-code Elimination, Straightening,**
 - If Simplification, Value Numbering,**
 - Copy Propagation, (SCCP) Sparse Conditional**
 - Constant Propagation, Common Subexpression**
 - Elimination, Loop-invariant Code Hoisting.**







A good intermediate code (IC) should have the following qualities:

- It should be relatively simple to generate from a syntax tree, otherwise its introduction would be too costly and error prone.**
- It should be relatively simple to generate a target code from the IC for the same reasons.**
- The semantics of the IC must be simple, clear, and unambiguous, so the optimization of the IC can be clearly specified and implemented.**

- **The syntax and semantics of the IC must be significantly less complex than that of the source code.**

So, what IC?

The simple answer is that IC is whatever a compiler designers decide to use.

Traditionally, there are three main approaches:

- ***Graphical representation* -- usually in the form of trees or graphs (often in the form of simplified syntax trees (e.g., A.W. Appel and his compilers for Tiger)).**

- ***Stack-machine code*** -- Java bytecode is an example of such an approach.
- ***Three-address code*** -- rudimentary Assembly-like instructions with two operands (hence two addresses) and a place to store the result (that is the third address).

In this talk we are using as the IC a three-address code developed for MACS.


```
class Factorial {
  shared int fact(int n) {
    int i, res;
    if (n == 0) return 1;
    for(res=1,i=2; i <= n; res=res*i,i++);
    return res;
  }
}
```

MACS source code of method fact ()

ic-start

...

Fact:

sec-start

arg n {val}{int}

t10 = 0

t12 = n == t10

t13 = !t12

if t13 goto L1

t14 = 1

return t14

L1:nop

L2:t13 = 1

```
L2:t13 = 1
    res = t13
    t14 = 2
    i = t14
    t15 = i <= n
    t16 = !t15
    if t16 goto L3
```

```
L4:nop
    t17 = res * i
    res = t17
    t18 = i + 1
    i = t18
    goto L2
```

```
L3:nop
```

```
L3:nop  
    return res  
sec-end  
...  
ic-end
```

A raw MIC code of method `fact()`

ic-start

...

fact:

sec-start

arg n {val}{int}

t12 = n == 0

t13 = !t12

if t13 goto L2

return 1

L2:res = 1 *(does this def. reach L3?)*

i = 2

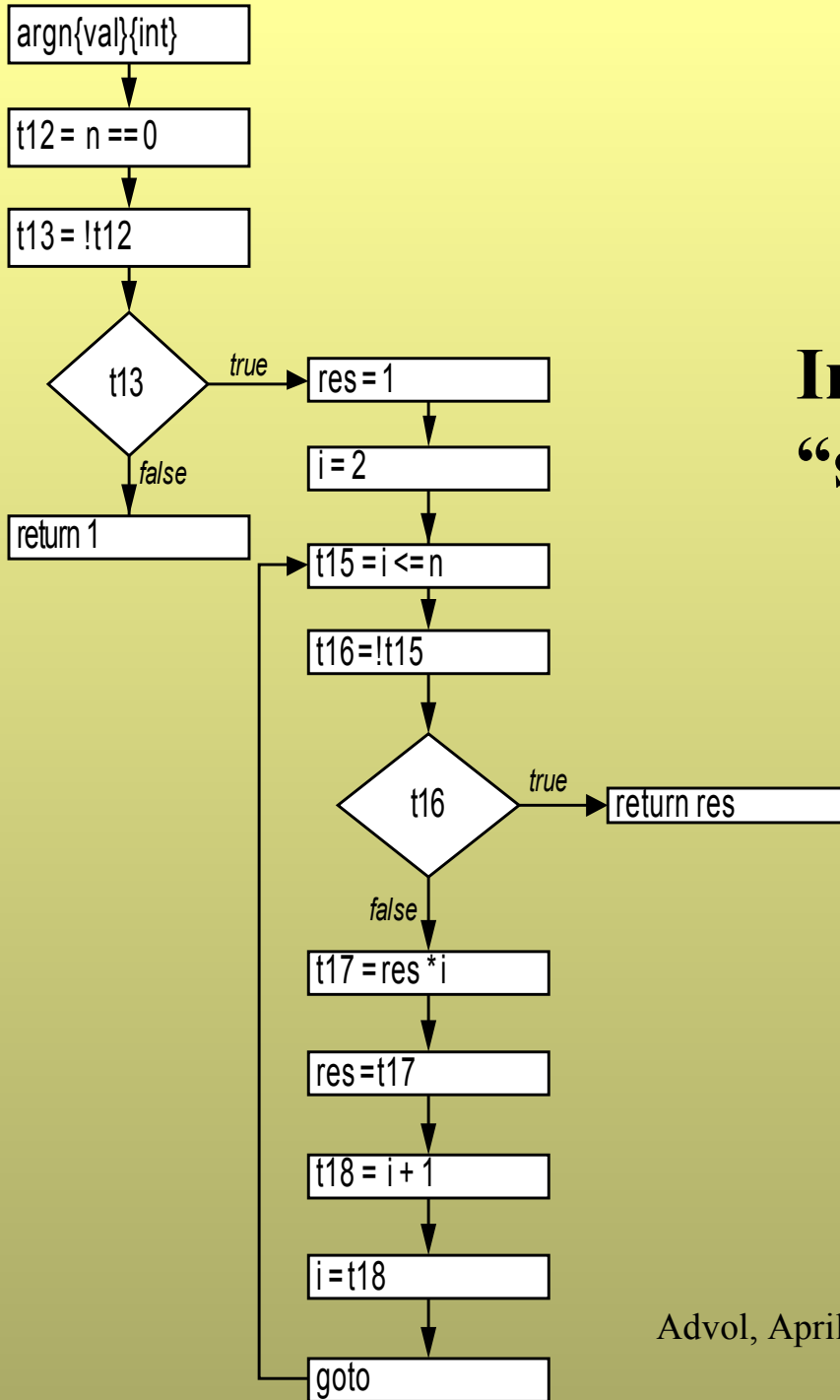
t15 = i <= n

t16 = !t15

```
    t16 = !t15
    if t16 goto L3
L4:t17 = res * i
    res = t17
    t18 = i + 1
    i = t18
    goto L2
L3:return res
sec-end
...
ic-end
```

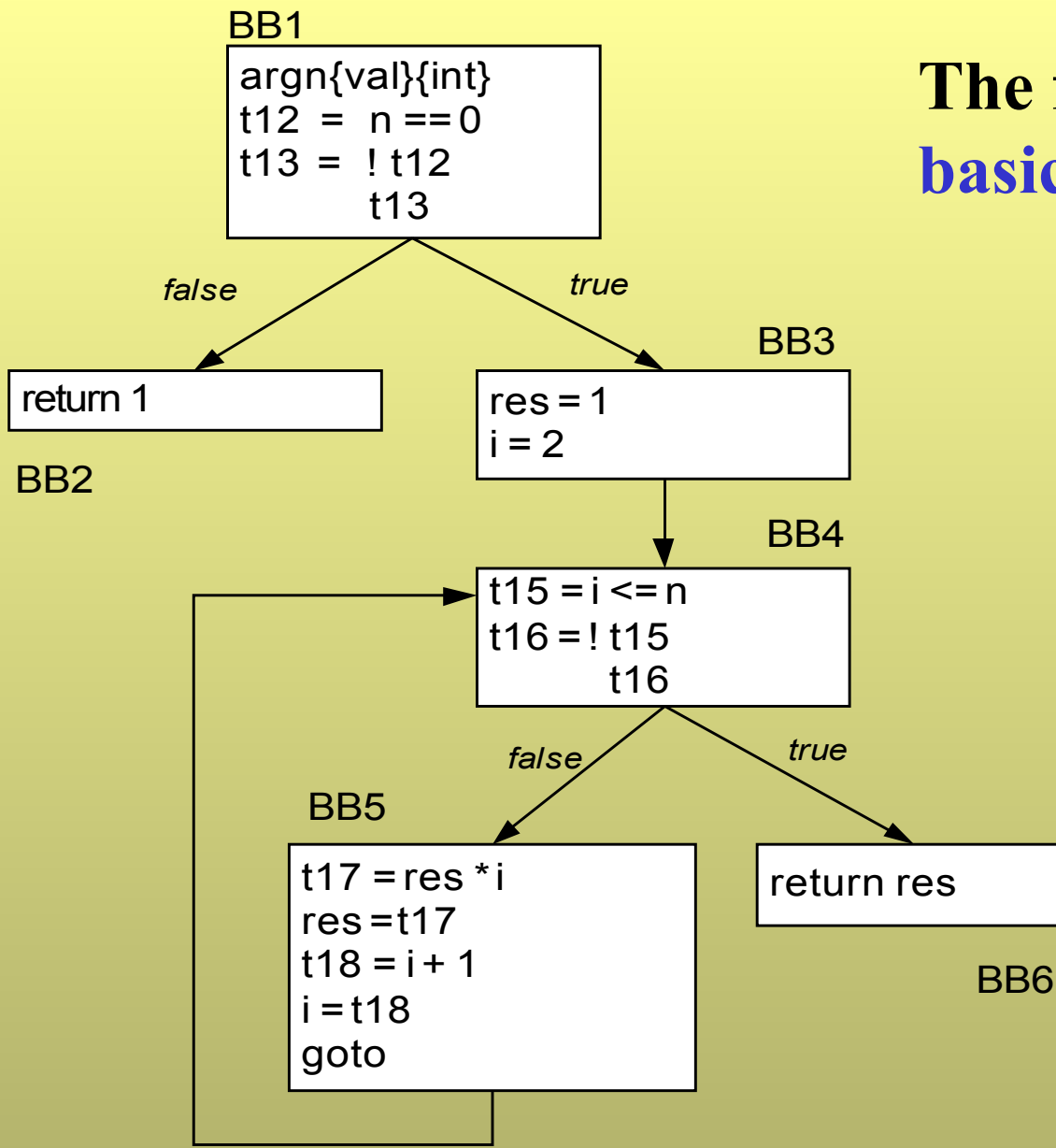
A streamlined MIC code of method `fact()`

It is hard to “see”, for instance, the loop in the IC code.

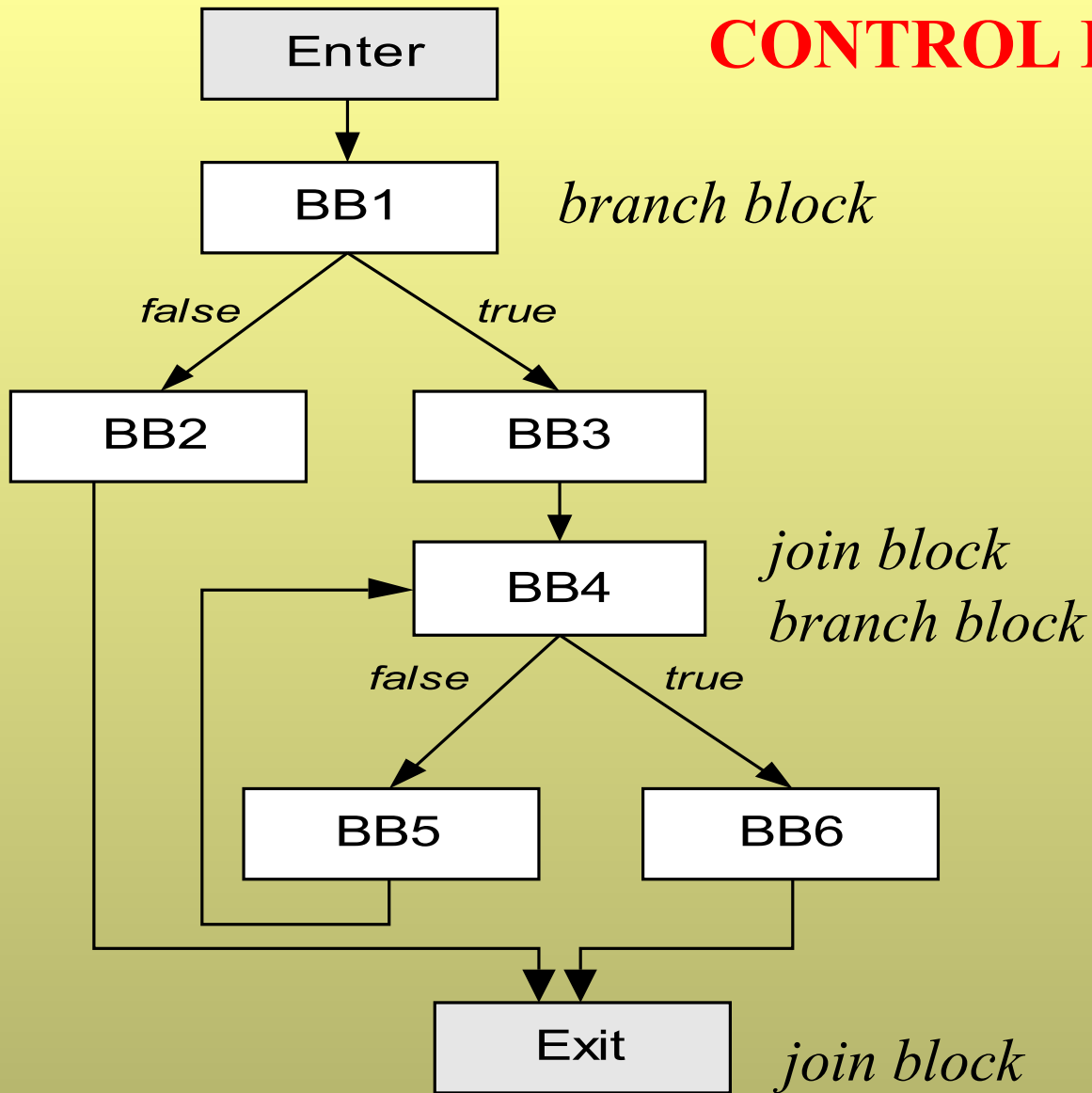


In the flowchart, we can “see” the loop again

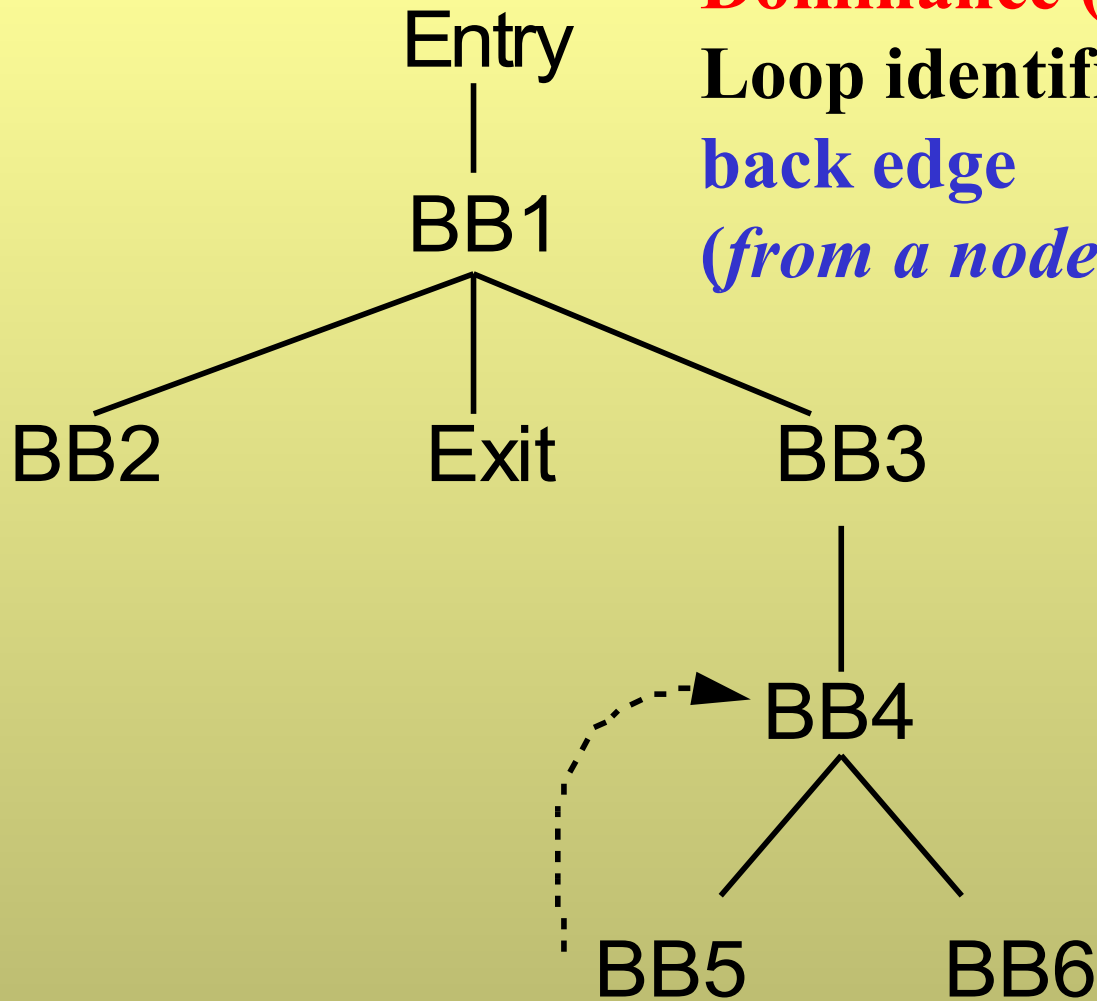
The flowchart with basic blocks



CONTROL FLOW GRAPH



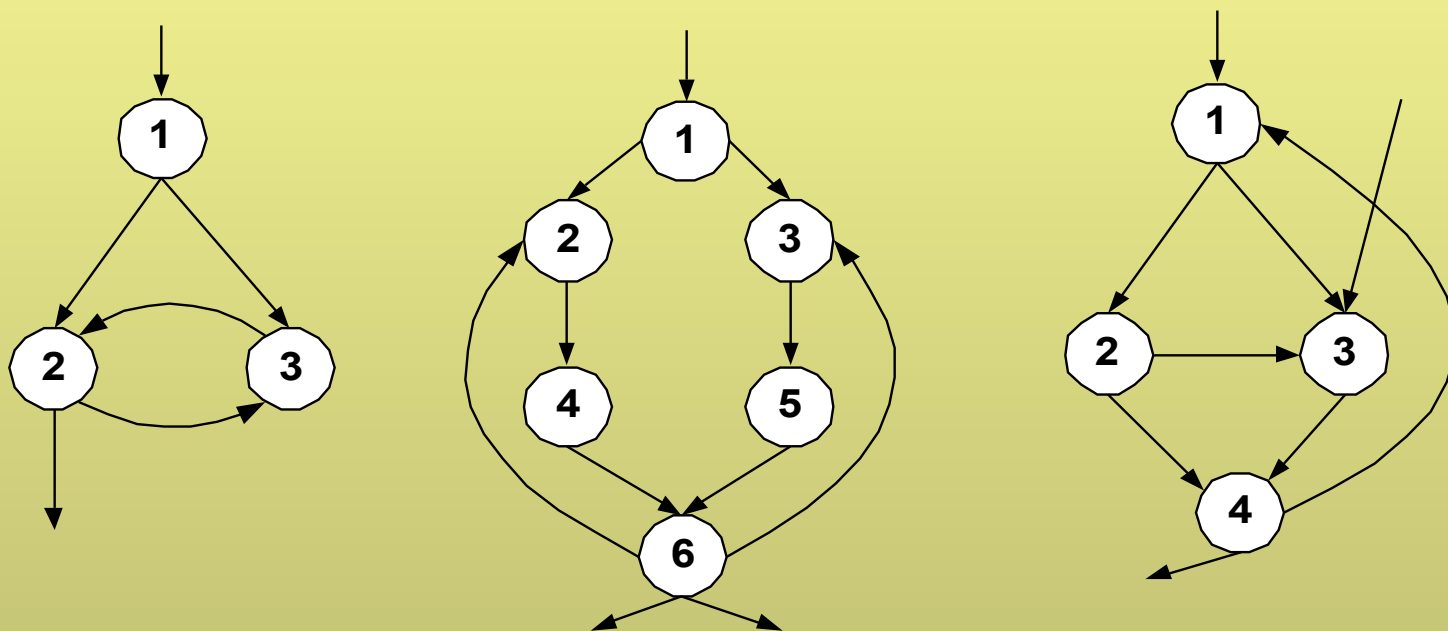
Dominance (dominator) tree
Loop identification via
back edge
(from a node to its dominator)



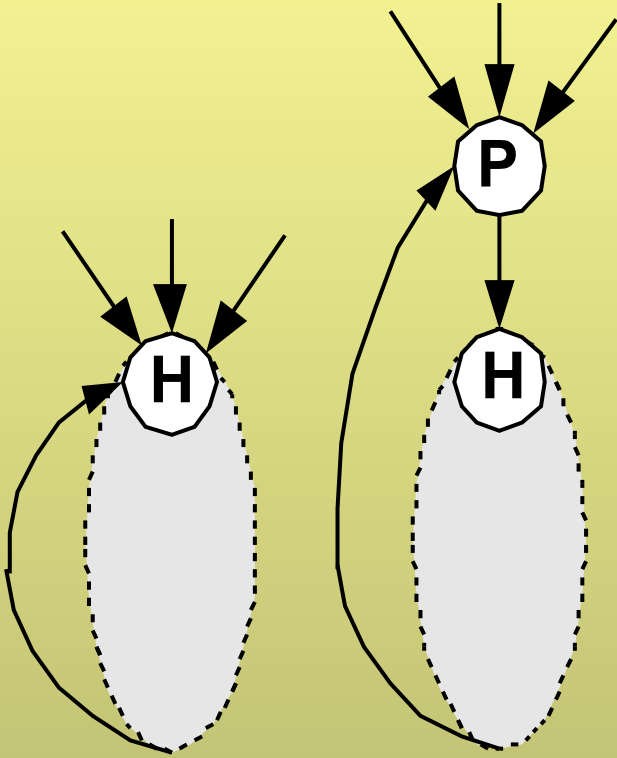
Calculations of dominators:

- **by a simple depth-first recursive algorithm based on the inductive definition of domination:**
A dominates B if
 - (a) $A=B$, or**
 - (b) A is a unique immediate predecessor of B, or**
 - (c) B has more than one immediate predecessor, and for every C immediate predecessor of B, A dominates C.**
- **More efficient algorithms are due to Lengauer+Tarjan and Alstrup+Lauridsen**

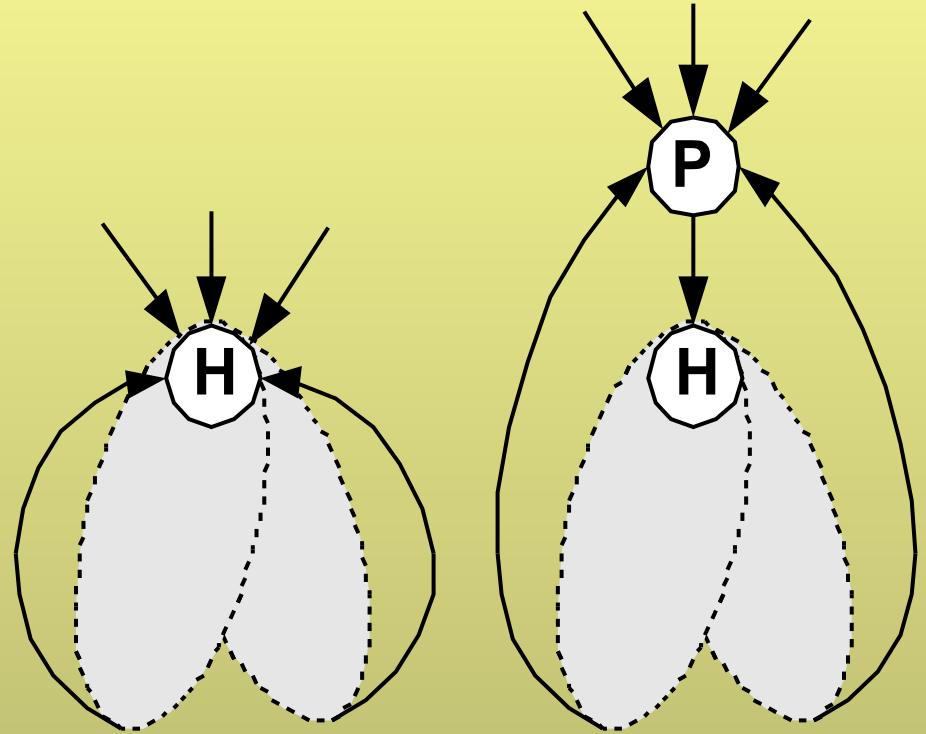
Dominators used for loop identification (to identify the back edge) and for a fast transformation to SSA.



Non-loops with edges going back -- lacking domination



Introducing a preheader P to a natural loop with a header H



Introducing a preheader P to two natural loops sharing a header H

Constant folding is a typical example of data-flow analysis. Only by reasoning about the flow, we can replace t_{12} by 5 in

$$t_{10} = 2$$

...

$$t_{11} = 3$$

...

$$t_{12} = t_{10} + t_{11}$$

Terminology: *definition* (defining assignment)

use

a definition kills a subsequent definition

Problem of reaching definitions - data-flow equations

$$Rd_{en}(i) = \bigcup_{j \in Pred(i)} Rd_{ex}(j)$$

$$Rd_{ex}(i) = (Ad(i) \cup Rd_{en}(i)) - Kill(i)$$

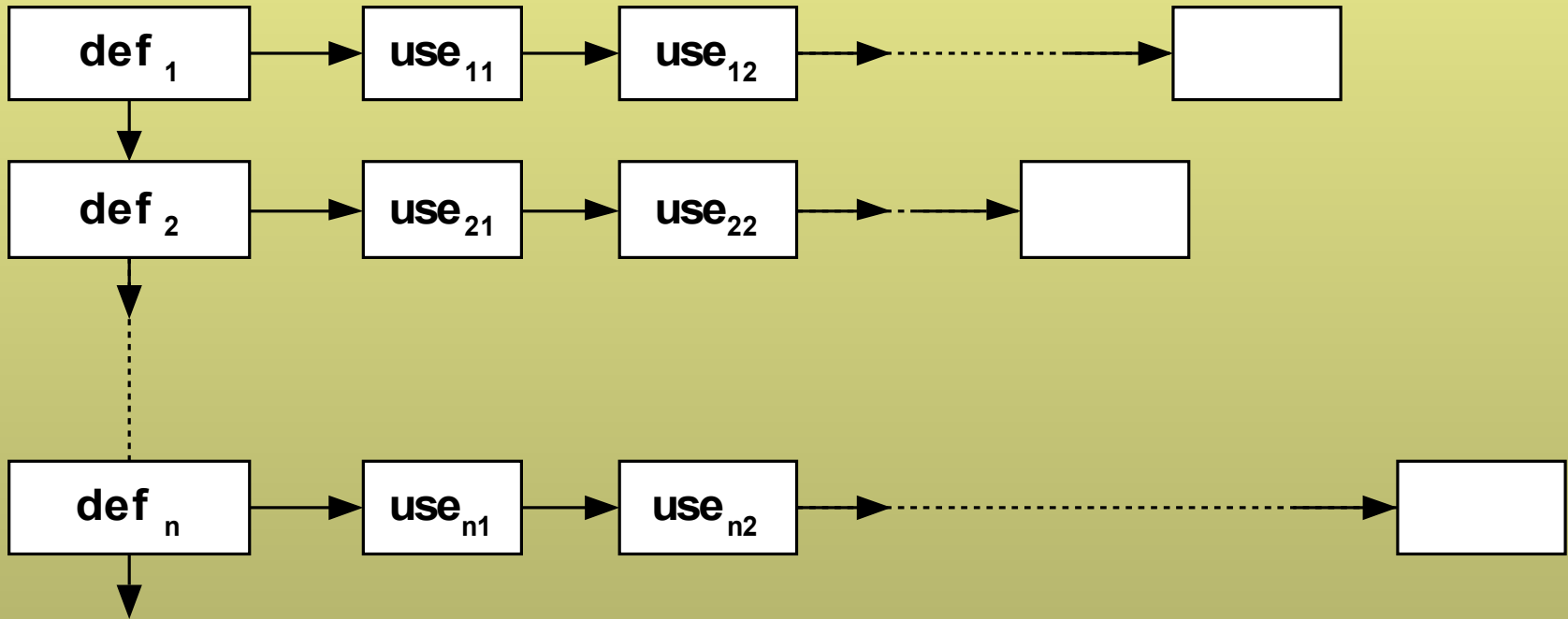
$$Ad(Enter) = Kill(Enter) = Rd_{en}(Enter) = Rd_{ex}(Enter) = \emptyset$$

- **Computing $Ad(i)$ - traverse the block i and put in $Ad(i)$ any definition encountered.**
- **Computing $Kill(i)$ - After computing all $Ad(i)$, traverse the block i and for every assignment encountered, put in $Kill(i)$ any definition from $\bigcup Ad(i)$ that has the same left-hand side variable (except the current assignment, it does not kill itself).**

The equations are now solved iteratively traversing the flow graph in breadth-first fashion.

Sparse representations:

- **du-chains and ud-chains**
- **SSA (static single assignment)**



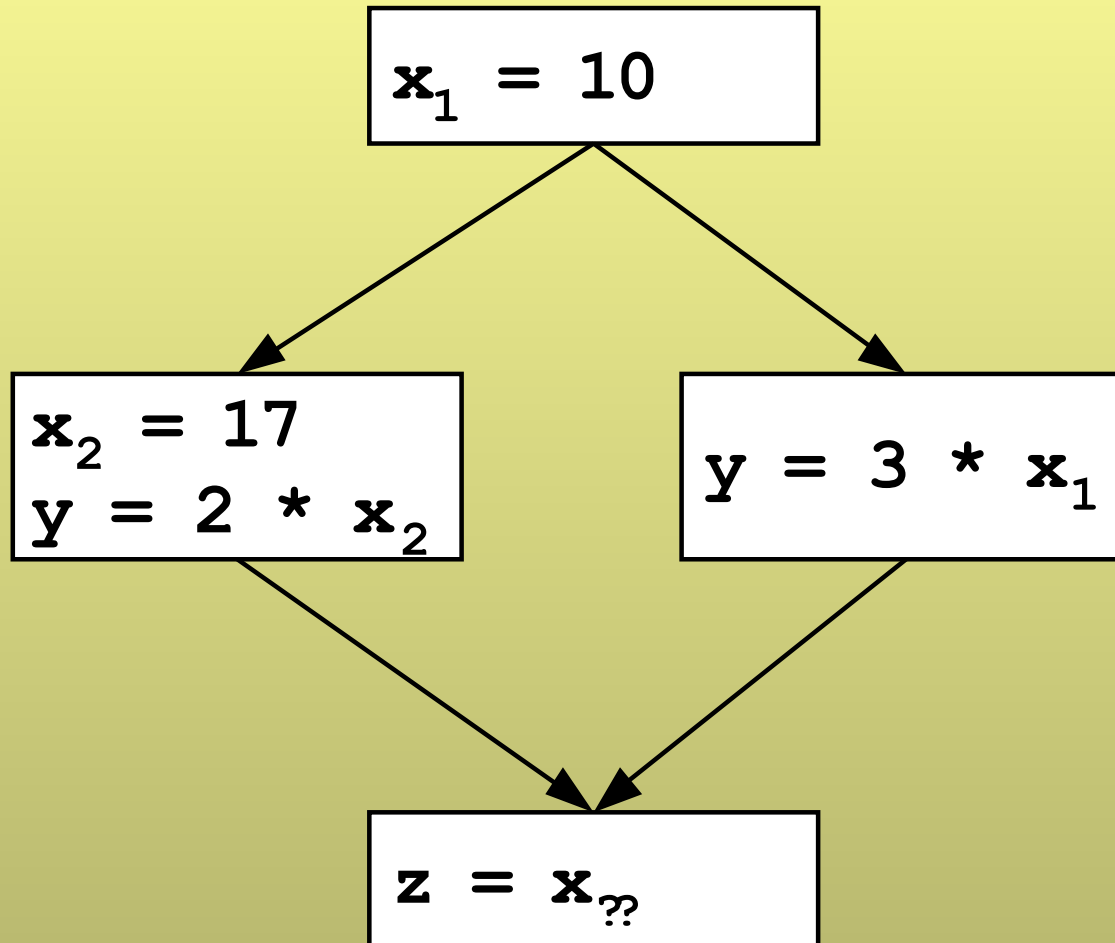
SSA can be regarded as systematic renaming of variables.

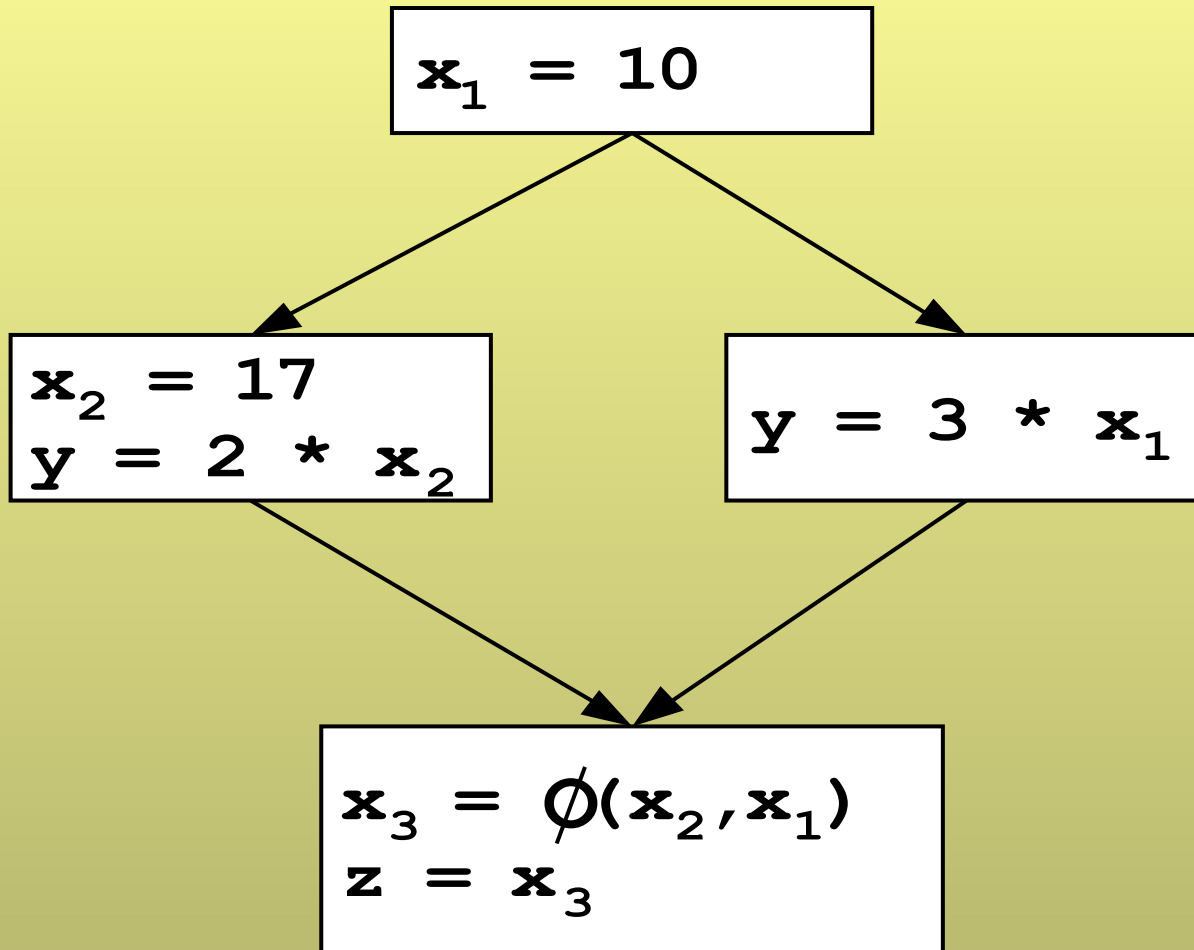
```
x = 3
y = x + 1
...
x = 10
...
z = 2 * x
```

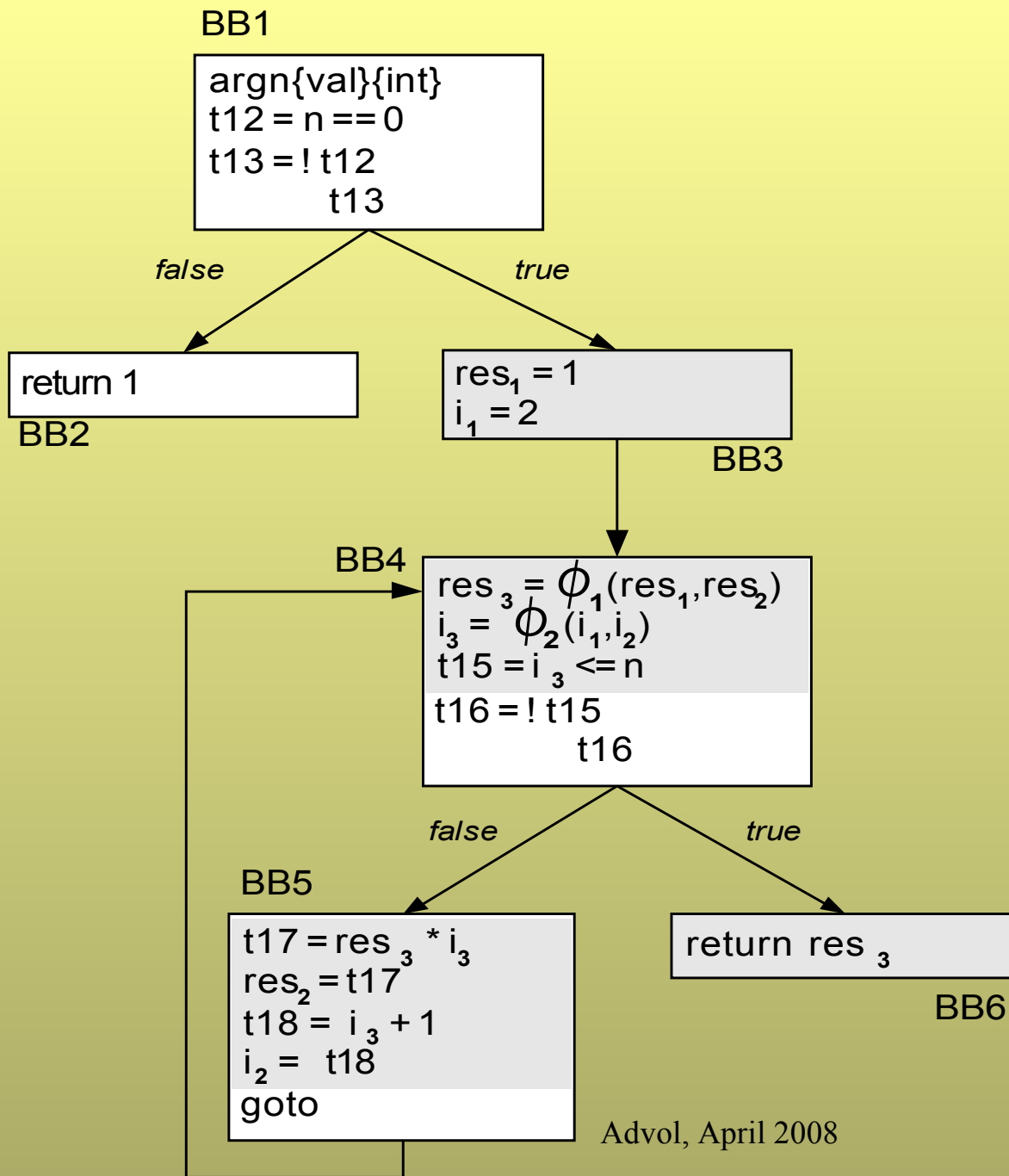
```
x1 = 3
y = x1 + 1
...
x2 = 10
...
z = 2 * x2
```

Not a problem within a simple basic block.

A problem in a join block.







Do we have to rename all variables ?

B1

```
...  
 $x_1 = 10$   
 $y = 20$   
...
```

No, y need not be renamed.

B2

```
...  
 $x_2 = 30$   
....
```

B3

```
 $x_3 = \phi(x_1, x_2)$   
 $z = y * x_3$   
...
```

Path-convergence criteria for insertion of Φ

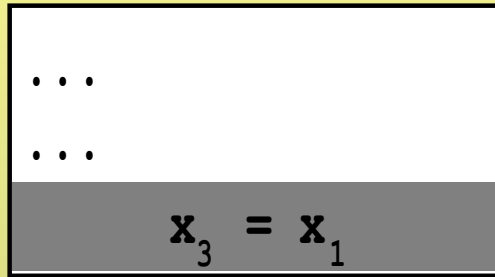
1. there are two distinct blocks B1 and B2 that both contain a definition of x that reaches to J, and
2. there is a path p_1 from B1 to J and a path p_2 from B2 to J that have no node in common except J, and
3. J can appear either in p_1 or in p_2 as an intermediate node (not the ending node), but not in both.

Numerous advantages for data analysis if the program is in SSA

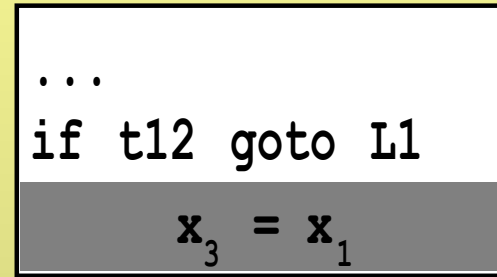
- **simplifies the analysis**
- **is almost linear in space (unlike do- and ud-chains)**
- **disassociates the parts of the code where the use of variable are just coincidental**

How to convert it back -- removing Φ

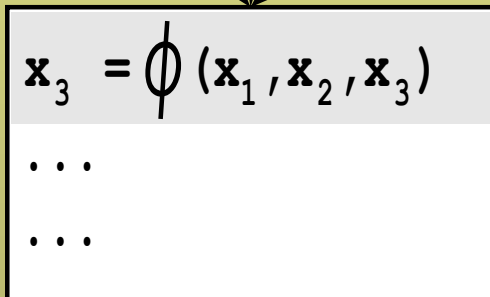
B1



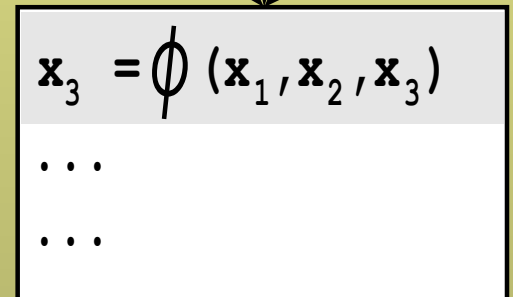
B1



B2

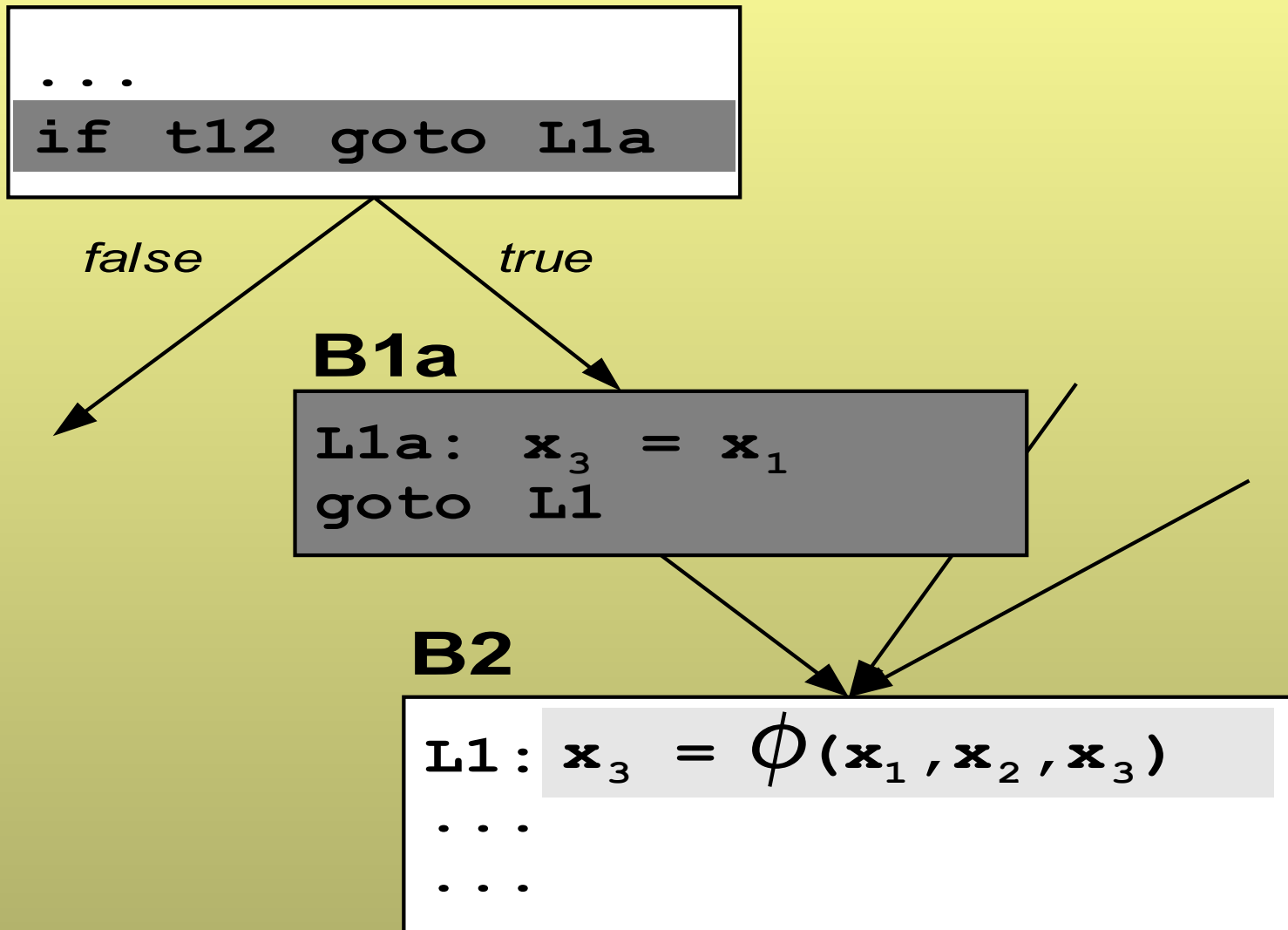


B2



Replacing Φ with no edge splitting

B1 Replacing Φ with edge splitting



Peephole optimization

```
...  
t17 = 2  
f4 = t17  
...
```

```
...  
f4 = 2  
...
```

Very typical (in MACS `Icgen()`)

```
...  
<label>:nop  
<instruction>  
...
```

```
...  
<label>:<instruction>  
...
```

Peephole optimization vs. Delayed code emission

Dead-code elimination

...

$t12 = x$

$x = t12 + 1$

...

no further use of x

x is dead (not used after definition), while $t12$ is not.

An instruction is dead if it only computes values not used in any instruction on any executable path leading from the instruction.

The dead-code elimination is an optimization method focused on detecting and eliminating dead instructions. It can be run on any level of IC but it is most suitable and effective on low level code including the target code.

A real care must be taken in evaluation of dead instructions; some real-world machine or assembly instructions are executed for their side-effects (like setting a condition code) and thus should not be considered dead even if they compute a dead value.

The basic approach to the detection of dead instructions is optimistic -- on a pass, mark some instructions as essential, and iterate the process in order to find the maximal set of essential instructions. The instructions that are not essential are considered dead and, consequently, eliminated.

Before starting the process of detection of essential instructions for the whole program, we identify the initial set of essential instructions by some other means, for instance for MIC -- I/O's, alloc(), transfer assignment, strstore(), etc. are deemed essential.

Unreachable-code elimination

```
    if (x | !x)
        goto A
    else
        goto B
    ...
A: ...
B: ...
```

Common error -- B will never be executed
clearly “visible” as $(x \mid !x)$ is a tautology

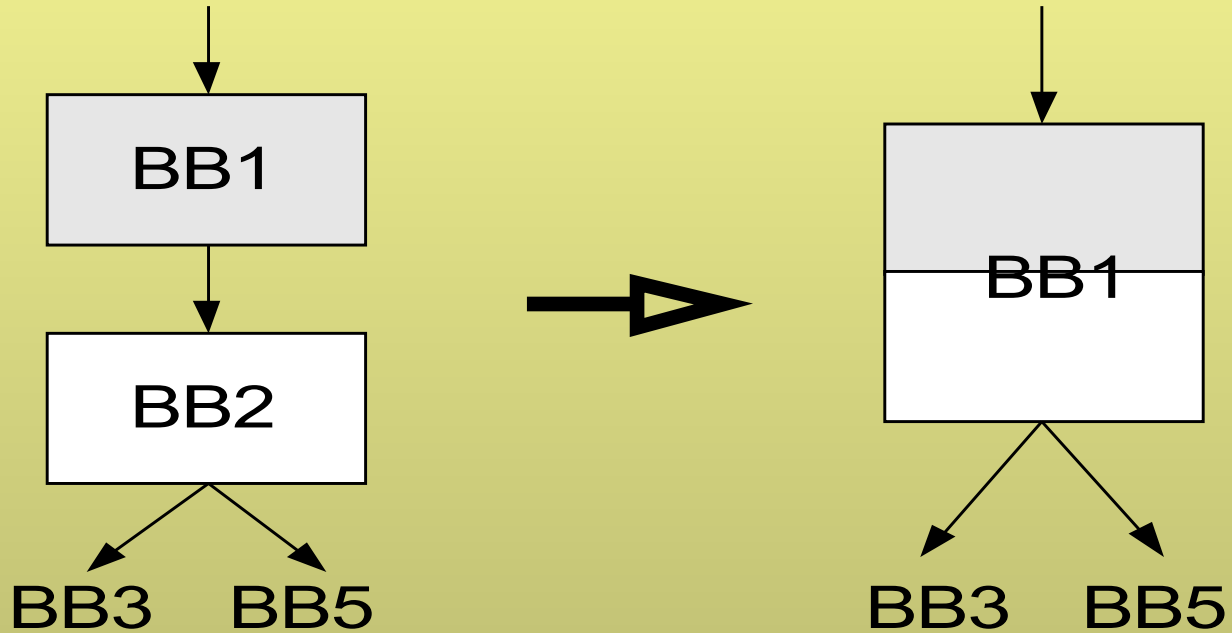
No longer “visible” in IC

```
    . . .  
    t13 = !x  
    t14 = x or t13  
    . . .  
    t15 = !t14  
    if t15 goto L1  
    goto A  
    goto L2  
L1: goto B  
L2: nop  
    . . .  
A: . . .  
B: . . .
```

The elimination of the unreachable code does not improve the performance as we are eliminating code that is NOT executed under any conditions. It can be run on any level of IC or target code.

In the graph-theoretical terms, the elimination of unreachable blocks boils down to the identifying the largest connected component of CFG containing the Entry block.

Straightening



If simplification

Example 1

```
if true goto L1      goto L1
if false goto L1
```

Example 2

```
    t12 = !t11
    if t12 goto L1
    goto L2
L1:...                if t11 goto L2
...                  L1:...
L2:...                ...
                    L2:...

```

Example 3

```
    if t12 goto L1
    ...
    goto L2
L1:goto L2
L2:...                if t12 goto L2
                    ...
                    L2:...

```

Value numbering

```
10      x  = y + t0
11      y  = x * z
12      t1 = y + t0
13      z1  = x * z
```

Clearly, we can simplify it to

```
10      x  = y + t0
11      y  = x * z
12      t1 = y + t0
13      z1  = y
```

Can we replace $t1=y+t0$ with x ? NO

Value numbering is a procedure that discerns what can and what cannot be replaced.

Value numbering in a basic block -- the basic idea is based on hashing -- if two expressions hash to the same, one can replace the other.

4 3 2 1

$$10 \quad x = y + t0$$

7 4 6 5

Hence we can replace $z1 = x * z$ with $z1 = y$

8 7 2 1

$$12 \quad t1 = y + t0$$

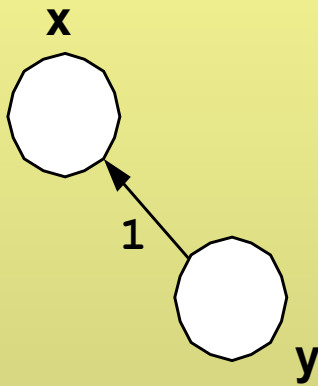
More important and more complex is global value numbering (global means in a procedure). This process is simplified by having the code for the procedure in SSA.

```
..      x1 = ...  
..      y1 = ...  
10     x2 = y1 + t0  
11     y2 = x2 * z  
12     t1 = y2 + t0  
13     z1 = x2 * z
```

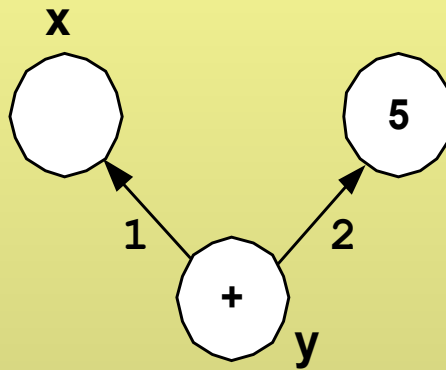
Note that in SSA form it is clear that we can replace $z_1 = x * z$ with $z_1 = y$



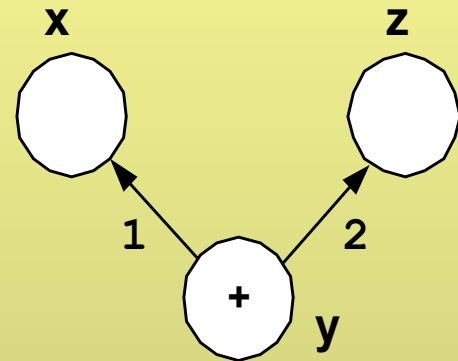
$$y = 4$$



$$y = x$$

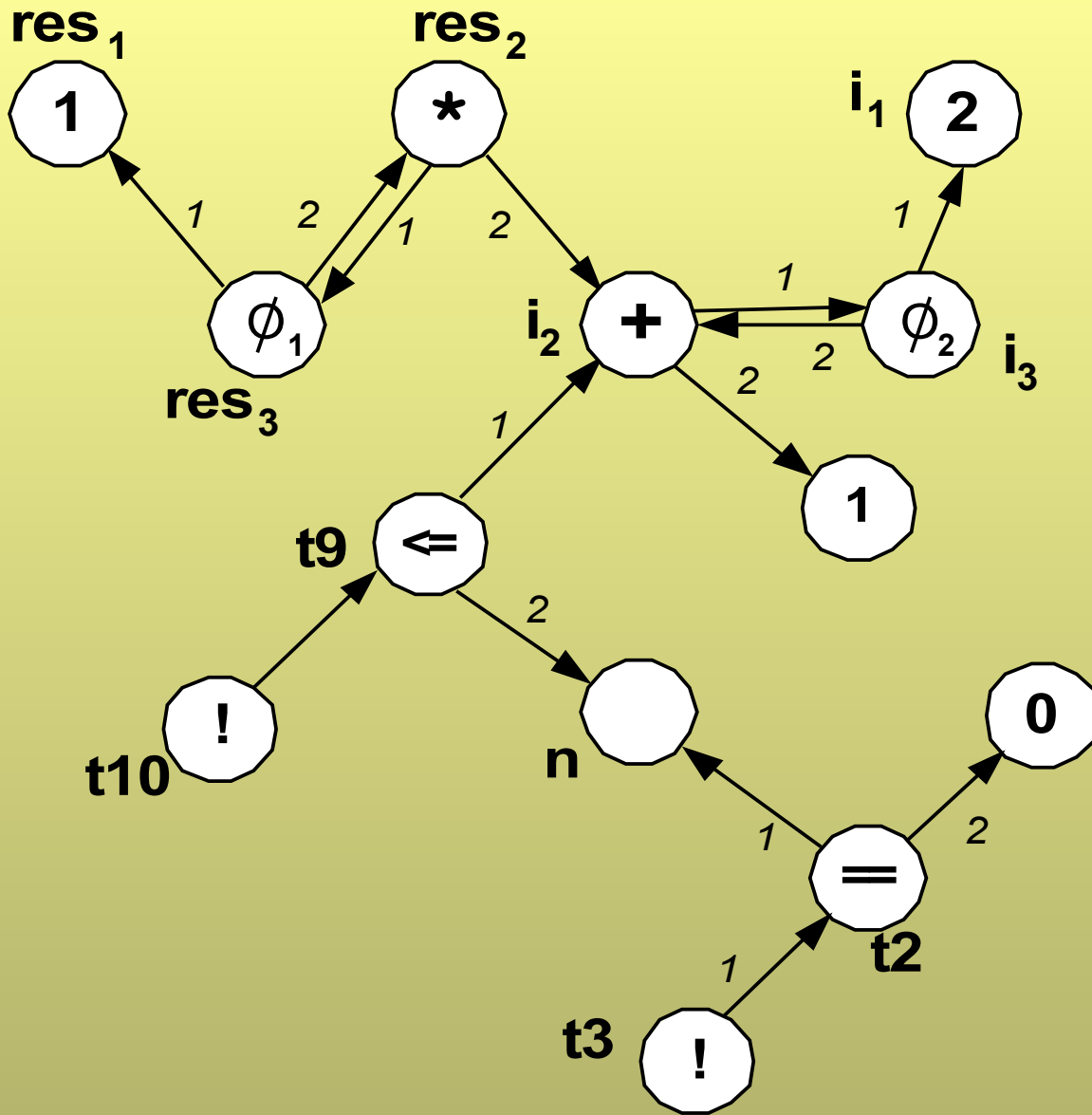


$$y = x + 5$$



$$y = x + z$$

Building a value graph



**Value graph
for `fact()`**

The basic idea is to define a congruence (indicating similarity) between variables and propagate it.

The *congruence* is a maximal relation on the value graph such that

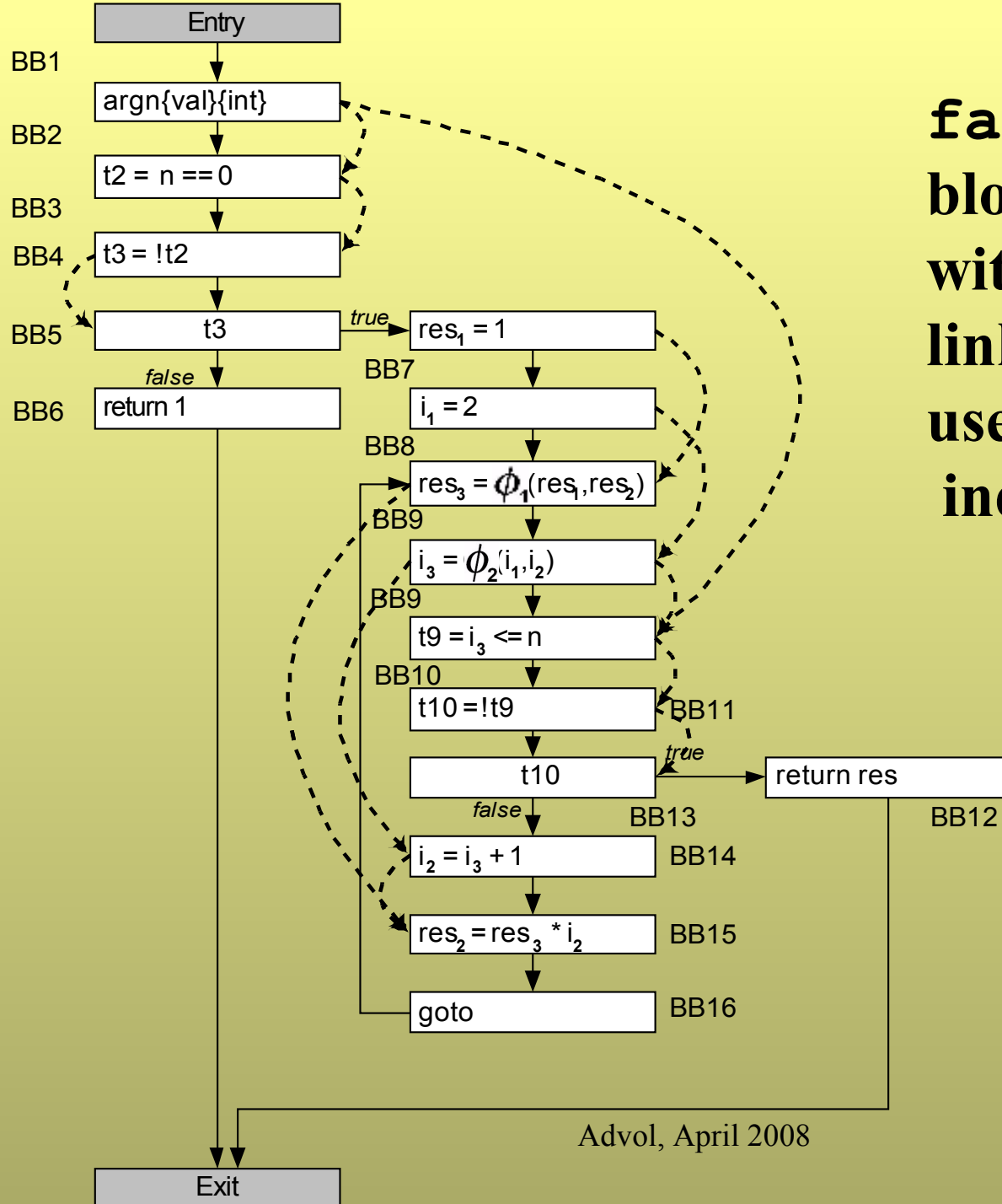
1. every node is *congruent* with itself,
2. two nodes labeled by the same constant are *congruent*,
3. two nodes with the same label are *congruent* if they have the same operators and the operators are *congruent*.

Typically, Aho, Hopcroft, Ullman's Partitioning Algorithm is used.

Copy propagation (or Copy folding)

Copy assignment $x = y$ means that we can replace a use of x by a use of y .

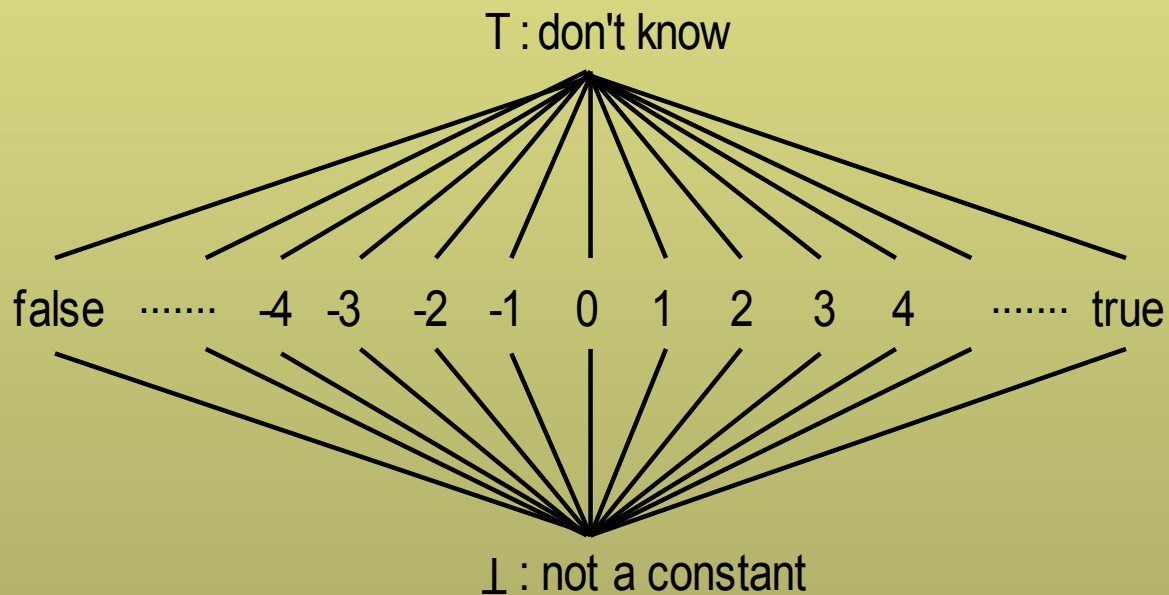
To do copy propagation, the program need not be in SSA form. If it is, it is straightforward as every variable has just a single defining assignment. Thus, when $x = y$ is encountered, y 's defining assignment had been encountered before, and this is the defining assignment for x , so it is guaranteed that neither y nor x can change in a later instruction.



fact () is single block SSA form with SSA edges linking defs with uses indicated by ----

Sparse Conditional Constant Propagation

Deals with redundancy of the form $x = \text{const}$ when we replace every use of x with const . We work with SSA form (sparse). It takes a form of symbolic computation using Constant Propagation Lattice.



t0 = 2

t1 = 3

t2 = t1+t2

t0 = 2

t1 = 3

t2 = 5

Can always be performed for integers and booleans

The same is not true for reals as the machine running the compiler may have a different floating-point arithmetic than the target machine. The problem of numerous exceptions must be considered. In simple terms, *the effort to include a constant expression evaluation in the compiler is just too extensive and generally, real constant folding is neither considered nor recommended.*

**There are many computational rules for CPL,
just for an illustration:**

The rules for CPL concerning \wedge are:

$$a \wedge \top = a$$

$$a \wedge \perp = \perp$$

$$c_1 \wedge c_2 = c_1, \text{ if } c_1 = c_2$$

$$c_1 \wedge c_2 = \perp, \text{ if } c_1 \neq c_2$$

The rule for ϕ : $\phi(a_1, a_2, \dots, a_n) = a_1 \wedge a_2 \wedge \dots \wedge a_n$.

```
i = 2;  
j = i * 3;  
if (j + 1 > 2)  
    i = 0;  
else  
    i = 1;  
k = 5 + i;
```

A careful examination of the code will reveal that it can be reduced to

```
j = 6;  
i = 0;  
k = 5;
```

After peephole optimization, the streamlined code:

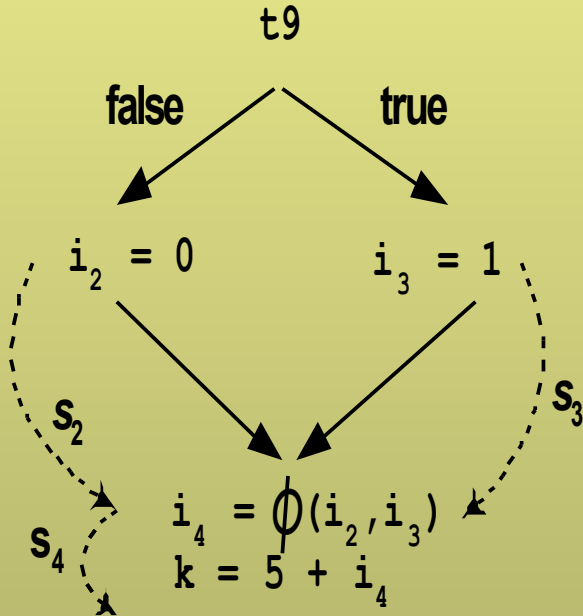
```
    i = 2
    t1 = i
    j = i * 3
    t6 = j + 1
    t8 = t6 > 2
    t9 = !t8
    if t9==true goto L2
L1: i = 0
    goto L3
L2: i = 1
L3: k = 5 + i
```

```

i1 = 2
t1 = i1
j = i1 * 3
t6 = j + 1
t8 = t6 > 2
t9 = !t8

```

- 1) evaluates **i** to 2 and adds the SSA edge **S**
- 2) evaluates **t1** to 2 and adds an SSA edge (not shown)
- 3) evaluates **j** to 6 and adds an SSA edge (not shown)
- 4) evaluates **t6** to 7 and adds an SSA edge (not shown)
- 5) evaluates **t8** to **true** and adds an SSA edge (not shown)
- 6) evaluates **t9** to **false** and adds an SSA edge (not shown)
- 7) marks the **false** branch as executable and leaves the **true** branch as non-executable



- 8) evaluates **i** to 0 and adds the SSA edge **S**



Common-subexpression elimination

```
t12 = x * y
```

```
t13 = x * y
```

can be replaced by

```
t14 = x * y
```

```
t12 = t14
```

```
t13 = t14
```

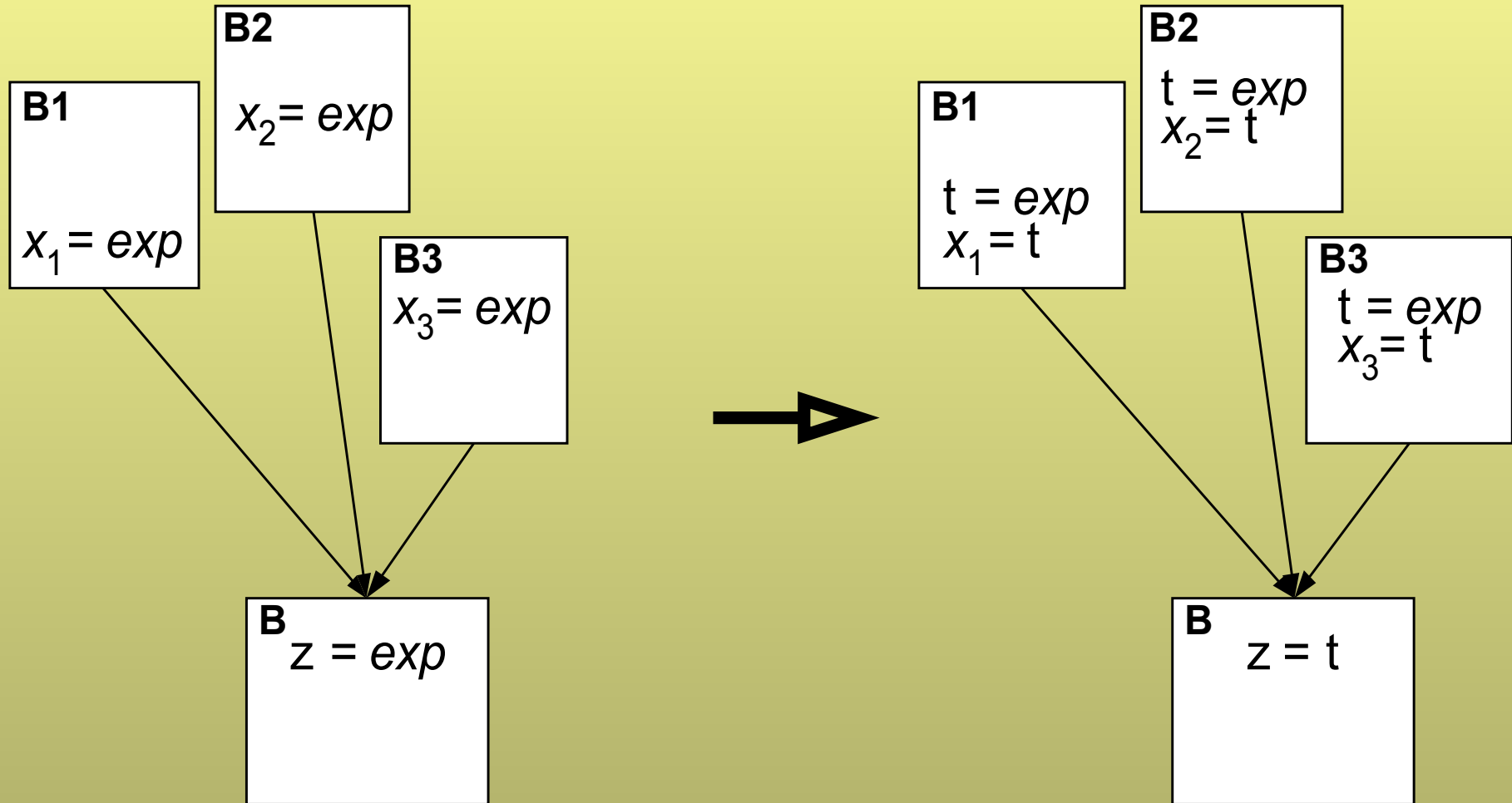
provided that the values of **x** and **y** have not changed between **t12 = x * y** and **t13 = x * y**.

For a basic block it is simple (similar ideas as in value numbering).

More involved is global elimination -- computing *available expressions*.

An expression *exp* is available at the entry to a basic block **B if on every path in the flow graph from the **Entry** block to **B** there is an evaluation of *exp* that is not subsequently killed by having one or both of its operands reassigned (hence the value of *exp* stays the same).**

Available expression and global common-subexpression elimination



We have to solve data flow equations

$$Avail_{en}(i) = \bigcap_{j \in Pred(i)} Avail_{ex}(j)$$

$$Avail_{ex}(i) = Avail(i) \cup (Avail_{en}(i) - Kill(i))$$

The purpose of all of this computation is to obtain $Avail_{en}(i)$ for all i .

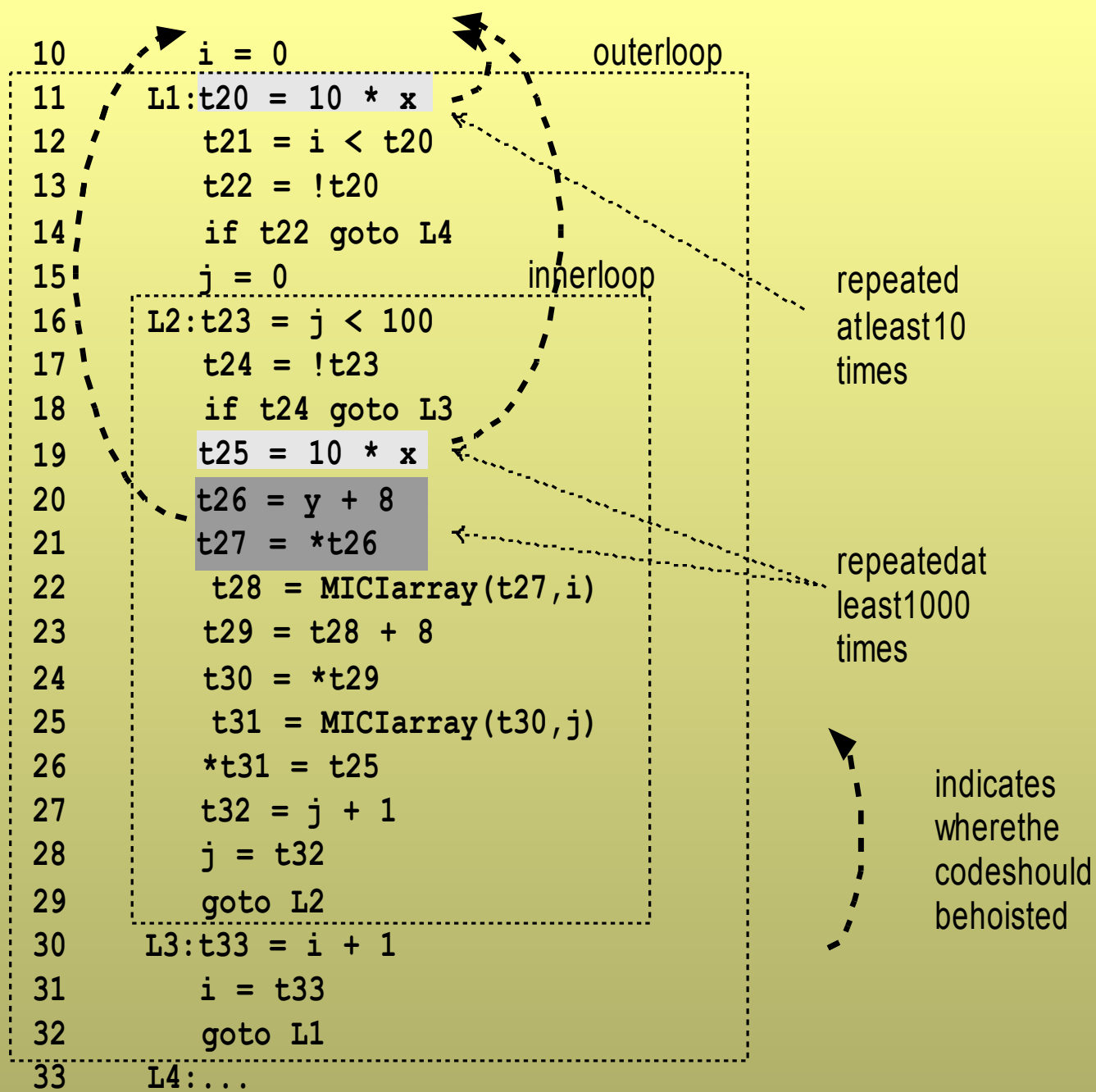
Loop-invariant code hoisting

```
...  
for(i = 0; i < 10 * x; i++)  
    for(j = 0; j < 100; j++)  
        y[i][j] = 10 * x;  
...
```

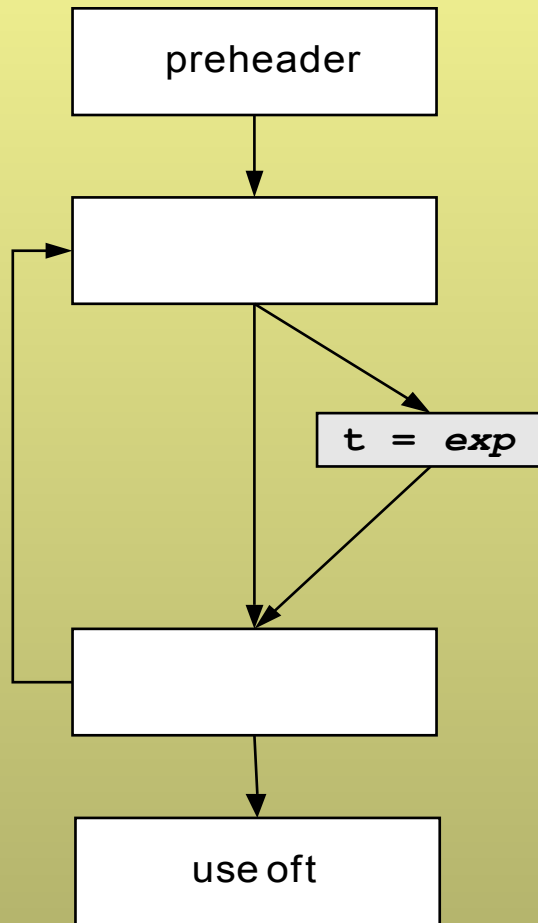
bad

```
...  
z = 10 * x;  
for(i = 0; i < z; i++)  
    for(j = 0; j < 100; j++)  
        y[i][j] = 10 * x;  
...
```

**Better
looks OK**

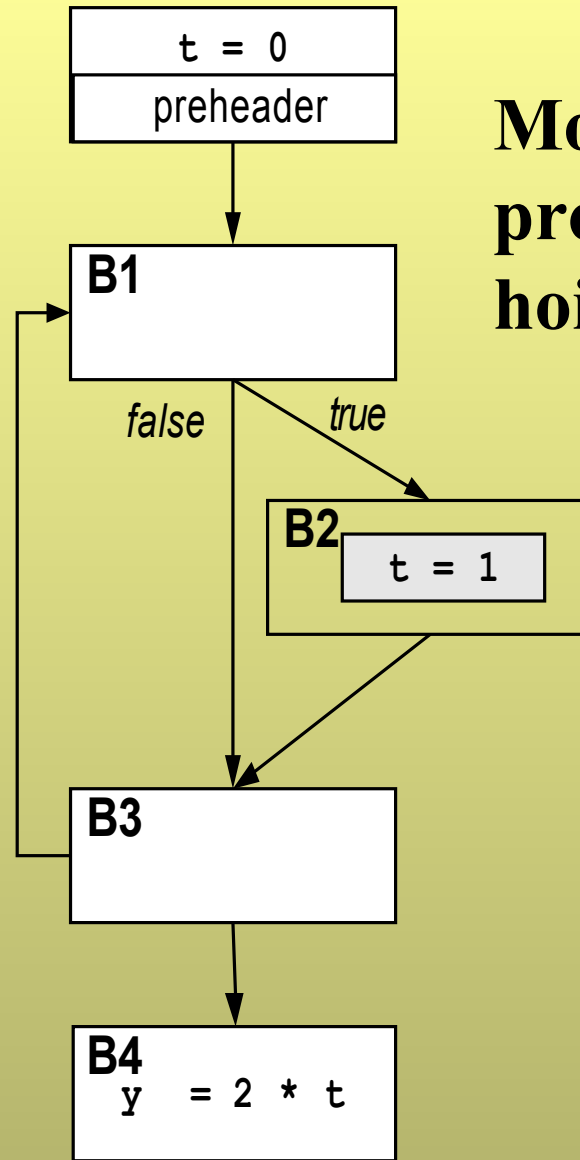
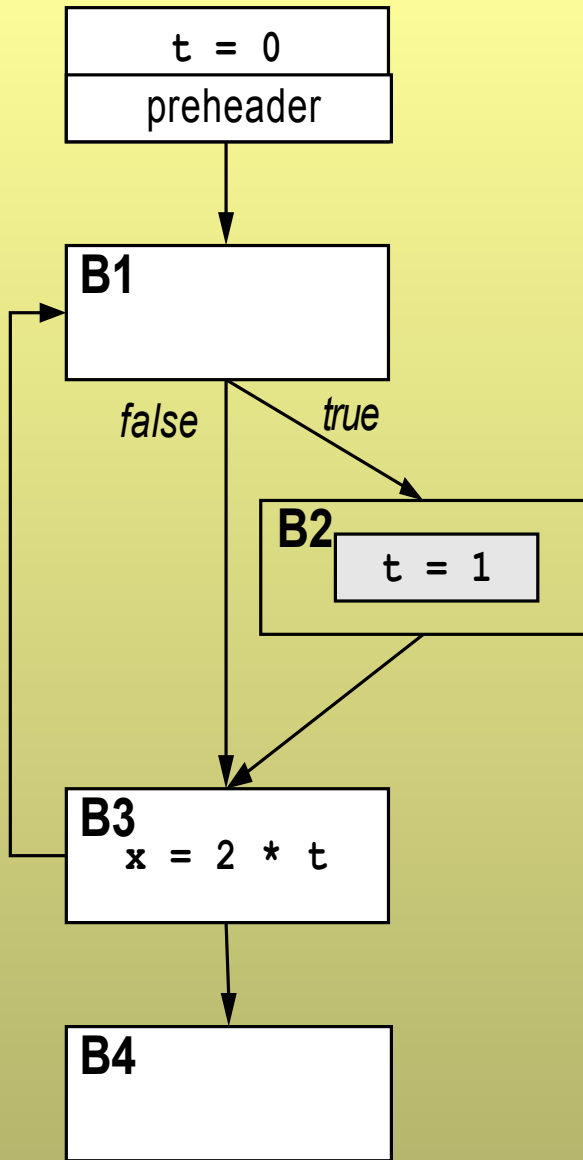


Definition of invariant is inductive, so again we work with a fix-point problem.



**Problem with
'simple' hoisting**

**still better to
hoist than not.**



**More complex
problem with
hoisting**

An assignment that is a candidate to be hoisted to the preheader, must satisfy additional constraints:

- **The assignment must be in a basic block that dominates all uses of the left-hand variable in the loop (this takes care of the situation depicted on the left as B2 does not dominate B3 with the use of t).**
- **The assignment must be in a basic block that dominates all exit blocks of the loop (this takes care to the situation depicted on the right as B2 does not dominate the exit block B3).**

on MACS source code level

Constantfolding
Algebraic simplification

on IC level

Peephole optimization
Dead-code elimination
Unreachable-code elimination
Straightening
If simplification

Constantfolding

Global value numbering
Local + global copy propagation
Sparse conditional constant propagation
Common-subexpression elimination
Loop-invariant code hoisting

Constantfolding

on MIC level

Peephole optimization
Register allocation

Peephole optimization
Dead-code elimination
Unreachable-code elimination
Straightening
If simplification

Constantfolding

Global value numbering
Local + global copy propagation
Sparse conditional constant propagation
Common-subexpression elimination
Loop-invariant code hoisting

Constantfolding

Thank you!